ON THE OBSERVABILITY AND CONTROLLABILITY CONDITION OF A TEMPERATURE MEASURING SYSTEM

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Abstract

In this paper, we discuss that the necessary and safficient condition for observability and controllability of a temperature measuring system connected with a proportional approximately derivative action type compensator to improve the dynamic error, where it is assumed that the characteristic of detecting element is delayed in first order.

As a result, it was found that the observability condition is $\alpha_c \neq \alpha_d$, $\kappa_c < \infty$, $\alpha_d \neq 0$, $\alpha_o \neq 0$, $C_1 \neq 0$, $C_2 \neq 0$, and the controllability condition is $\alpha_c \neq \alpha_o$, $\kappa_d \neq 0$, $\kappa_c < \infty$, $\alpha_c \neq 0$, $\alpha_o \neq 0$, for two outputs observing system.

1. Introduction

The transfer function of a detecting part in the temperature measuring system is often approximated at the first order delay system. Hence, if the corner frequency of a detector is lower than the frequency which the true temperature varies, a dynamic error occurs in this system.

For the method which compensates the delay of the detector, the proportional plus incomplete derivative action (PD for short) has been used. For the study of a compensator using PD action, in 1950 Ziegler and Nichols accomplished compensating the time lag of a gas enclosure type temperature detector by a pneumatic PD action[1)], and Sawaragi, Wada et al. using a quartz bar and a steel bar for the temperature detector, gave the PD compensation pneumatically by the differencial of their expansion[2)]. For electric PD compensators, types using a CR circuit plus a chopper-amplifier[3)] and a DC amplifier[4)5)] have been developped.

To discuss the observability and controllability[6)] on the basis of the modern control theory for these PD compensators is significant, although cases which do not necessarily satisfy the observability or controllability condition exist[7)]. So we discuss the necessary and sufficient conditions for which the above PD compensators hold good observability and controllability.

2. Notations

E_i 〔°C〕 : true temperature (input to the temperature measurement system)

E_d 〔 V 〕 : measured temperature

E_o 〔 V 〕 : output of the compensator (compensation output of temperature measurement system)

I 〔 A 〕 : output of the resistance to current converter (R_t/I)

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K_d [mA/°C] : gain constant of the detector
K_c [mA/V] : gain constant of the compensator
T_d [s] : time constant of the detecting element
T_c [s] : main time constant of the compensator
T_o [s] : subordinate time constant of the compensator (time constant of the imcomplete derivation)
$G_d(S)$ [mA/°C] : transfer function of the detecting element
$G_c(S)$ [V/mA] : transfer function of the compensating element
$\kappa_d = K_d / T_d$: gain coefficient of the detecting element
$\kappa_c = T_o K_c / (T_c + T_o)$: gain coefficient of the PD compensator
$\alpha_d = 1 / T_d$: coefficient of the time constant of the detecting element
$\alpha_c = 1/(T_c + T_0)$: coefficient of the main time constant of the compensator
$\alpha_0 = 1/T_0$: coefficient of the subordinate time constant of the compensator
C_1, C_2 : conversion coefficient of the observation instrument

3. Compensating method for the temperature detecting lag by PD action

For the compensating system of the detector in Fig. 1, if the transfer function of the detecting element

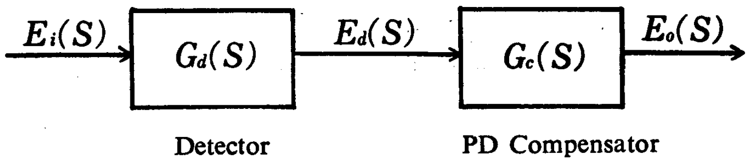

Fig.1 Block diagram of PD compensating system for temperature detector

$G_d(S)$ is approximated by a first order delayed system and the transfer function of the compensating element $G_c(S)$ is proportional plus imcompleted derivative action, the following equations are obtained

$$G_d(S) = \frac{K_d}{1 + T_d S} \tag{1}$$

$$G_c(S) = \frac{1}{K_c}\left(1 + \frac{T_c S}{1 + T_o S}\right) \tag{2}$$

Now, when to this system the step input voltage $E_i\, u(t)$ (with $u(t) = 1,\ t > 0$; $u(t) = 0,\ t < 0$) is applied, if the conditions $T_o \ll T_d$, $T_c = T_d$, $K_c = K_d$ exist, the output voltage $E_o(t)$ is given by the following equation

$$E_o(t) \fallingdotseq E_i \{1 - \exp(-t/T_o)\} \tag{3}$$

If this system has no compensator, the output voltage $E_o(t)$ for $K_d = 1$ is

$$E_o(t) = E_i \{1 - \exp(-t/T_d)\} \tag{4}$$

Then the time constant T_d of the detecting element is improved to T_o by the compensating element $G_c(S)$.

4. Design and conditions of the compensator under consideration of observability and controllability

In this section we will discuss the system at general terms. When the scaler input corresponding to the true temperature is $u(t)$ and the state variable of the system $x(t)$ is a second order vector, then b becomes the 2 nd order in the state equation $dx(t)/dt = Ax(t) + bu(t)$, and the system in Fig. 2 is organized.

From Fig. 2, the following state equations and observation equations are obtained

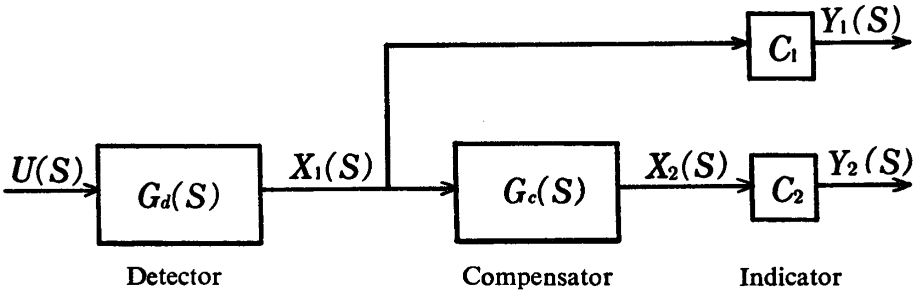

Fig.2 Block diagram of a compensating system under consideration of observability and controllability

$$\left.\begin{aligned} X_1(S) &= G_d(S)\cdot U(S) \\ X_2(S) &= G_c(S)\cdot X_1(S) \end{aligned}\right\} \tag{5}$$

$$Y_1(S) = c_1X_1(S), \quad Y_2(S) = c_2X_2(S) \tag{6}$$

and with respect to $G_d(S)$, $G_c(S)$

$$G_d(S) = \frac{K_d}{1+T_d S} = \frac{\kappa_d}{S+\alpha_d} \tag{7}$$

$$\begin{aligned} G_c(S) &= \frac{1}{K_c}\left(1+\frac{T_c S}{1+T_o S}\right) \\ &= \frac{1}{K_c}\,\frac{(T_o+T_c)S+1}{1+T_o S} \\ &= \frac{1}{\kappa_c}\,\frac{S+\alpha_c}{S+\alpha_o} \end{aligned} \tag{8}$$

where

$$\left.\begin{aligned} \frac{1}{\kappa_c} &= \frac{T_o+T_c}{T_o K_c}, & \kappa_d &= \frac{K_d}{T_d} \\ \alpha_c &= \frac{1}{T_o+T_c}, & \alpha_o &= \frac{1}{T_o} \\ \alpha_d &= \frac{1}{T_d}, & T_c &\gg T_o \end{aligned}\right\} \tag{9}$$

Substituting the equations (7), (8) into the equations (5), (6), the following equations are obtained.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\alpha_d & 0 \\ (\alpha_c-\alpha_d)/\kappa_c & -\alpha_o \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \kappa_c \\ \kappa_d/\kappa_c \end{bmatrix} u \tag{10}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{11}$$

Therefore, the state equation and the observation equation are

$$\left.\begin{aligned} \frac{dx(t)}{dt} &= Ax(t) + bu(t) \\ y(t) &= Cx(t) \end{aligned}\right\} \tag{12}$$

where

$$\left.\begin{aligned} x(t) &= \text{col}\,(x_1(t),\ x_2(t)) \\ y(t) &= \text{col}\,(y_1(t),\ y_2(t)) \end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned} A &= \begin{bmatrix} \alpha_d & 0 \\ (\alpha_c-\alpha_d)/\kappa_c & -\alpha_o \end{bmatrix} \\ b &= \begin{bmatrix} \kappa_d \\ \kappa_d/\kappa_c \end{bmatrix} \\ C &= \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \end{aligned}\right\} \tag{14}$$

Fig. 3 is obtained from the equations (7), (8).

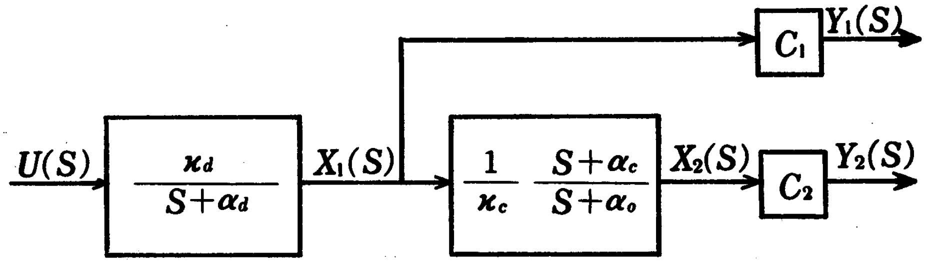

Fig.3 Block diagram of a detecting element and it's PD compensating system

The necessary and sufficient condition for observability of the system in Fig. 3 is

$$\text{rank}\,[C' \vdots A'C'] = 2 \tag{15}$$

where $'$ denotes the transposition. From equation (14),

$$C' = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \tag{16}$$

$$A'C' = \begin{bmatrix} -\alpha_d & (\alpha_c - \alpha_d)/\kappa_c \\ 0 & -\alpha_o \end{bmatrix} \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}$$

$$= \begin{bmatrix} -\alpha_d\, c_1 & c_2\,(\alpha_c - \alpha_d)\,/\,\kappa_c \\ 0 & -\alpha_o\, c_2 \end{bmatrix} \tag{17}$$

Consequently, the condition which satisfy the following equation

$$\text{rank} \left[\begin{array}{cc|cc} c_1 & 0 & -\alpha_d c_1 & (\alpha_c - \alpha_d)c_2/\kappa_c \\ 0 & c_2 & 0 & -\alpha_0 c_2 \end{array}\right] = 2, \tag{18}$$

are $\alpha_c \neq \alpha_d$, $\kappa_c < \infty$, $\alpha_d \neq 0$, $\alpha_o \neq 0$, $c_1 \neq 0$, $c_2 \neq 0$ (19)

Next, the necessary and sufficient condition for controllability of the system in Fig. 3 is

$$\text{rank}\,[b \vdots Ab] = 2 \tag{20}$$

From equation (14),

$$Ab = \begin{bmatrix} -\alpha_d & 0 \\ (\alpha_c - \alpha_d)/\kappa_c & -\alpha_o \end{bmatrix} \begin{bmatrix} \kappa_d \\ \kappa_d/\kappa_c \end{bmatrix}$$

$$= \begin{bmatrix} -\alpha_d\,\kappa_d \\ (\alpha_c - \alpha_o - \alpha_d)\,\kappa_d\,/\,\kappa_c \end{bmatrix} \tag{21}$$

Therefore, the conditions which satisfy the following equation

$$\text{rank} \begin{bmatrix} \kappa_d & -\alpha_d\,\kappa_d \\ \kappa_d\,/\,\kappa_c & (\alpha_c - \alpha_o - \alpha_d)\,\kappa_d\,/\,\kappa_c \end{bmatrix} = 2, \tag{22}$$

are $\alpha_c \neq \alpha_o$, $\kappa_d \neq 0$, $\kappa_c < \infty$, $\alpha_c \neq 0$, $\alpha_o \neq 0$ (23)

5. Conclusion

(1) For a PD compensating system which has been organized as a one input and one output system, the system organization shown in Fig. 3 satisfies observability and controllability mathematically and physically.

(2) We can process data by means of modern analytical methods.

(3) The values of the coefficients for the electric PD compensator are

$\alpha_d = 0.05\ s^{-1}$, $\kappa_d = 0.053$ V/°C

$c_1 = 18.75$ °C/V, $\alpha_c = 0.0435\ s^{-1}$

$\kappa_c = 0.053$ °C/V, $\alpha_o = 0.3333\ s^{-1}$

$c_2 = 18.75$ °C/V

These values satisfy all conditions of the equations (19), (20).

REFERENCE

1) J. G. Ziegler and N. B. Nichols: "Dynamic Accuracy in Temperature Measurement", Instruments, 23, P.60 (1960).

2) Sawaragi, Wada, Sadamoto: "On A Pneumatic PD Type Temperature Detector", Trans. of JAACE, **5**—3, 151/156 (1961).

3) Kaneda: "Measurement of Gas Temperature in an Internal Combustion Engine", The 14th Seminar of JSME, 17/26 (1960).

4) Mine, Doi: "Time Delayless Detecting Method using Inverse Transfer Function Type Compensator (No. 4)", Bulletin of Ube Technical College, 5, P. 23 (1967. 3).

5) Mine, Doi, Konomi: "The Method of Approximately Inverse Transfer Function Type Compensation of The Dynamic Error of Detection on a Real Process", JIEE; Preprint of Kyushu Congress, P.11 (1967.11).

6) R. E. Kalman: "On The General Theory of Control Systems", Proc. of the First IFAC Moscow Congress, P. 481, Butterworth, (1961).

7) Y. Takahashi: "System and Control", P187, Iwanami (1968.9).

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