A Note on Closure Property of Sublogarithmic Space-Bounded 1-Inkdot Alternating Turing Machines with Only Existential (Universal) States

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Abstract

A 1-inkdot Turing machine is a slightly modified Turing machine model which has been introduced in order to show a strong separation between deterministic and nondeterministic complexity classes. An alternating Turing machine is a generalization of nondeterministic one, and is considered as a mechanism to model parallel computations. This paper investigates closure property of sublogarithmic space-bounded 1-inkdot alternating Turing machines with only universal (existential) states, and shows, for example, that for any function $L(n)$ such that $L(n) \geq \log \log n$ and $L(n) = o(\log n)$, the class of sets accepted by weakly (strongly) $L(n)$ space-bounded 1-inkdot two-way alternating Turing machines with only universal (existential) states is not closed under complementation, length-preserving homomorphism, concatenation with regular sets, and Kleene closure.

Key Words: alternating Turing machines, 1-inkdot Turing machines, sublogarithmic space, closure property, computational complexity

1. Introduction and Notations

Alternating Turing machines (ATM's) were introduced in Ref. 1) as a mechanism to model parallel computations. We assume that the reader is familiar with the basic concepts and terminology concerning ATM's and computational complexity (If necessary, see Refs. 1) — 4)).

A two-way ATM (2ATM) we consider here has a read-only input tape and a semi-infinite read-write worktape. We denote a 2ATM with only universal states (resp., existential states, i.e., a two-way nondeterministic Turing machine) by 2UTM (resp., 2NTM). Further, we denote by 2DTM a two-way deterministic Turing machine.

Ranjan et al. introduced in Ref. 2) a slightly modified Turing machine model, called a *1-inkdot Turing machine*, to show a strong separation of deterministic and nondeterministic complexity classes. The 1-inkdot Turing machine is a Turing machine with the additional power of marking at most 1 tape-cell in the input tape (with an inkdot). This tape-cell is marked once and for all (no erasing). The action of the machine depends on the current states, the input and the worktape symbols scanned currently, and the presence of the inkdot on the currently scanned tape-cell. For each X∈{A, U, N, D}, let $2XTM^*$ denote a 1-inkdot 2XTM.

For each X∈{A, U, N, D} and any function $L(n)$, *strong*-2XTM($L(n)$) and *weak*-2XTM($L(n)$) denote the classes of sets accepted by strongly and weakly $L(n)$ space-bounded 2XTM's, respectively, and *strong*-$2XTM^*(L(n))$ and *weak*-$2XTM^*(L(n))$ denote those accepted by strongly and weakly $L(n)$ space-bounded $2XTM^*$'s, respectively.

Ranjan et al. showed in Ref. 2) that for any two-way Turing machines with an inkdot, nondeterministic and deterministic sublogarithmic space complexity classes are not equal. After that, Geffert showed in Ref. 5) that *strong*-$2NTM^*(\log \log n)$ — *strong*-2NTM($o(\log n)$) $\neq \phi$, where from now on logarithms are base 2. In Ref. 6), Inoue et al. strengthened the result above and showed that *strong*-$2NTM^*(\log \log n)$ — *weak*-2NTM($o(\log n)$) $\neq \phi$. Inoue et al. also introduced $2ATM^*$ in Ref.7) as a generalization of $2NTM^*$

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and showed that for each X∈{A, U} and each Y∈{U, N}, *strong*-2XTM*(log log *n*) — *weak*-2XTM(*o*(log *n*)) $\neq \phi$ and *strong*-2ATM*(log log *n*) — *weak*-2YTM*(*o*(log *n*)) $\neq \phi$. Furthermore, Yosihnaga et al. introduced in Ref. 4) sublinear space-bounded 1-inkdot two-way alternating multi-counter automata and showed that the accepting power of these automata with only existential states are incomparable with the one of these automata with only universal states.

While several important properties of 1-inkdot Turing machines with sublogarithmic space are explicated up to now, there are little investigations relating to closure properties of these as we know. So, from the theoretical interest, we investigate in the present paper closure properties of 2UTM* and 2NTM* which have sublogarithmic space. We show, for example, that for each *m*∈{*strong*, *weak*}, each X∈{N, U} and any function log log *n* ≤ *L*(*n*) = *o*(log *n*), *m*-2XTM*(*L*(*n*)) is not closed under complementation, length-preserving homomorphism, concatenation with regular sets, and Kleene closure.

2. Results

Throughout this paper, let *L*(*n*) be a function such that *L*(*n*) ≥ log log *n* and *L*(*n*) = *o*(log *n*), and let *m* be an element of the set {*strong*, *weak*}.

For the standard Turing machines, in Ref. 3), it is shown that *weak*-2XTM(*L*(*n*)) is not closed under complementation, but it is unknown if *strong*-2XTM (*L*(*n*)) is closed under the operation, for each X ∈ {U, N}. Our corresponding result is shown as follows:

Theorem 2.1. *m*-2NTM*(*L*(*n*)) and *m*-2UTM*(*L*(*n*)) are not closed under complementation.

Proof. It is proved in Ref. 4) that the class of sets accepted by strongly (weakly) *S*(*n*) space-bounded 2-way 1-inkdot alternating multi-counter automata with only existential (universal) states is not closed under complementation, where *S*(*n*) is a function such that *S*(*n*) ≥ log *n* and log *S*(*n*) = *o*(log *n*). By using the same languages and idea as in Ref. 4), we can directly prove the theorem. □

Remark: It is trivial that *m*-2NTM(*L*(*n*)) and *m*-2NTM*(L(n)) (resp., *m*-2UTM(*L*(*n*)) and *m*-2UTM* (*L*(*n*))) are closed under union (resp., intersection). Thus, *m*-2ATM(*L*(*n*)) and *m*-2ATM*(*L*(*n*)) are closed under union and intersection.

For the standard Turing machines, in Ref. 3), it is shown that *m*-2NTM(*L*(*n*)) is closed under intersection, but whether *m*-2UTM(*L*(*n*)) is closed under union or not is an open problem. On the other hand, for the 1-inkdot Turing machines with *L*(*n*) space, different situations occur. To get our results, we need the following key lemma:

Lemma 2.1. Let

$$\begin{aligned} A = \{ & bin(1)\#bin(2)\#\ldots\#bin(n)\# \\ & wcw_{11}cw_{12}c\ldots cw_{1r_1}ccw_{21}cw_{22}c\ldots cw_{2r_2} \\ & \in \{0,1,c,\#\}^+ \mid n \geq 2 \ \&\ w \in \{0,1\}^{\lceil \log n \rceil} \ \& \\ & \forall k\,(1 \leq k \leq 2)[r_k \geq 1 \ \& \\ & \forall l\ (1 \leq l \leq r_k)[w_{kl} \in \{0,1\}^+]] \ \& \\ & \forall i\ (1 \leq i \leq 2)[\exists j\ (1 \leq j \leq r_i)[w = w_{ij}]]\}, \end{aligned}$$

and let

$$\begin{aligned} B = \{ & bin(1)\#bin(2)\#\ldots\#bin(n)\# \\ & wcw_{11}cw_{12}c\ldots cw_{1r_1}ccw_{21}cw_{22}c\ldots cw_{2r_2} \\ & \in \{0,1,c,\#\}^+ \mid n \geq 2 \ \&\ w \in \{0,1\}^{\lceil \log n \rceil} \ \& \\ & \forall k\,(1 \leq k \leq 2)[r_k \geq 1 \ \& \\ & \forall l\ (1 \leq l \leq r_k)[w_{kl} \in \{0,1\}^+]] \ \& \\ & \exists i\ (1 \leq i \leq 2)[\forall j\ (1 \leq j \leq r_i)[w \neq w_{ij}]]\}, \end{aligned}$$

where for each positive integer *m* ≥ 1, *bin*(*m*) denotes the string in $\{0, 1\}^+$ that represents the integer *m* in binary notation (with no leading zeros). Then,

(1) $A \notin$ *weak*-2NTM*(*L*(*n*)) and

(2) $B \notin$ *weak*-2UTM*(*L*(*n*)).

Proof. From the assertions in Refs. 8) and 9), we straightforwardly get (1) and (2), respectively. □

Theorem 2.2. *m*-2NTM*(*L*(*n*)) and *m*-2UTM*(*L*(*n*)) are not closed under intersection and union, respectively.

Proof. Let

$$\begin{aligned} A_1 = \{ & bin(1)\#bin(2)\#\ldots\#bin(n)\# \\ & wcw_{11}cw_{12}c\ldots cw_{1r_1}ccw_{21}cw_{22}c\ldots cw_{2r_2} \\ & \in \{0,1,c,\#\}^+ \mid n \geq 2 \ \&\ w \in \{0,1\}^{\lceil \log n \rceil} \ \& \\ & \forall k\,(1 \leq k \leq 2)[r_k \geq 1 \ \& \\ & \forall l\ (1 \leq l \leq r_k)[w_{kl} \in \{0,1\}^+]] \ \& \\ & \exists j\ (1 \leq j \leq r_1)[w = w_{1j}]\}, \end{aligned}$$

$$\begin{aligned} A_2 = \{&bin(1)\#bin(2)\#\ldots\#bin(n)\# \\ &wcw_{11}cw_{12}c\ldots cw_{1r_1}ccw_{21}cw_{22}c\ldots cw_{2r_2} \\ &\in \{0,1,c,\#\}^+ \mid n \ge 2 \;\&\; w \in \{0,1\}^{\lceil \log n \rceil} \;\& \\ &\forall k\,(1 \le k \le 2)[r_k \ge 1 \;\& \\ &\forall l\,(1 \le l \le r_k)[w_{kl} \in \{0,1\}^+]] \;\& \\ &\exists j\,(1 \le j \le r_2)[w = w_{2j}]\}, \end{aligned}$$

$$\begin{aligned} B_1 = \{&bin(1)\#bin(2)\#\ldots\#bin(n)\# \\ &wcw_{11}cw_{12}c\ldots cw_{1r_1}ccw_{21}cw_{22}c\ldots cw_{2r_2} \\ &\in \{0,1,c,\#\}^+ \mid n \ge 2 \;\&\; w \in \{0,1\}^{\lceil \log n \rceil} \;\& \\ &\forall k\,(1 \le k \le 2)[r_k \ge 1 \;\& \\ &\forall l\,(1 \le l \le r_k)[w_{kl} \in \{0,1\}^+]] \;\& \\ &\forall j\,(1 \le j \le r_1)[w \ne w_{1j}]\}, \end{aligned}$$

and

$$\begin{aligned} B_2 = \{&bin(1)\#bin(2)\#\ldots\#bin(n)\# \\ &wcw_{11}cw_{12}c\ldots cw_{1r_1}ccw_{21}cw_{22}c\ldots cw_{2r_2} \\ &\in \{0,1,c,\#\}^+ \mid n \ge 2 \;\&\; w \in \{0,1\}^{\lceil \log n \rceil} \;\& \\ &\forall k\,(1 \le k \le 2)[r_k \ge 1 \;\& \\ &\forall l\,(1 \le l \le r_k)[w_{kl} \in \{0,1\}^+]] \;\& \\ &\forall j\,(1 \le j \le r_2)[w \ne w_{2j}]\}. \end{aligned}$$

It can be observed that

(i) both A_1 and A_2 are in *strong*-2NTM$^*(L(n))$,
(ii) both B_1 and B_2 are in *strong*-2UTM$^*(L(n))$,
(iii) $A_1 \cap A_2 = A$, and
(iv) $B_1 \cup B_2 = B$.

From these facts and Lemma 2.1, the present theorem follows. □

Theorems 2.1 and 2.2 above show closure properties of m-2NTM$^*(L(n))$ and m-2UTM$^*(L(n))$ under Boolean operations.

We will then discuss whether m-2NTM$^*(L(n))$ and m-2XTM$^*(L(n))$ are closed under the language operations: length-preserving homomorphism, concatenation with regular set and Kleene closure.

The following theorem shows non-closure under the language operations above for m-2NTM$^*(L(n))$:

Theorem 2.3. m-2NTM$^*(L(n))$ is not closed under
(1) length-preserving homomorphism,
(2) concatenation with regular sets, and
(3) Kleene closure.

Proof of (1): Let

$$\begin{aligned} A_3 = \{&bin(1)\#bin(2)\#\ldots\#bin(n)\# \\ &wcw_{11}cw_{12}c\ldots cw_{1r_1}cc_1w_{21}c_2w_{22}c_3\ldots c_{r_2}w_{2r_2} \\ &\in \{0,1,c,d,\#\}^+ \mid n \ge 2 \;\&\; w \in \{0,1\}^{\lceil \log n \rceil} \;\& \\ &\forall k\,(1 \le k \le 2)[r_k \ge 1 \;\& \\ &\forall l\,(1 \le l \le r_k)[w_{kl} \in \{0,1\}^+]] \;\& \\ &\exists i\,(1 \le j \le r_1)[w = w_{1j}] \;\& \\ &\exists j\,(1 \le j \le r_2)[c_j = d \;\&\; w = w_{2j} \;\& \\ &\forall p\,(1 \le p \le r_2,\, p \ne j)[c_p = c]]\}. \end{aligned}$$

We can easily show that $A_3 \in$ *strong*-2NTM$^*(L(n))$. Further, $h(A_3) = A$, where h is a length-preserving homomorphism such that $h(0) = 0$, $h(1) = 1$, $h(\#) = \#$, and $h(c) = h(d) = c$. From these facts and Lemma 2.1, (1) follows.

Proof of (2): Let

$$\begin{aligned} A_4 = \{&bin(1)\#bin(2)\#\ldots\#bin(n)\# \\ &wcw_{11}cw_{12}c\ldots cw_{1r_1}ccw_{21}cw_{22}c\ldots cw_{2r_2} \\ &\in \{0,1,c,\#\}^+ \mid n \ge 2 \;\&\; w \in \{0,1\}^{\lceil \log n \rceil} \;\& \\ &\forall k\,(1 \le k \le 2)[r_k \ge 1 \;\& \\ &\forall l\,(1 \le l \le r_k)[w_{kl} \in \{0,1\}^+]] \;\& \\ &\exists j\,(1 \le j \le r_1)[w = w_{1j}] \;\&\; w = w_{2r_2}\} \end{aligned}$$

and let $A_5 = \{cw \mid w \in \{0, 1\}^+\}^*$. It can be easily seen that

(i) $A_4 \in$ *strong*-2NTM$^*(L(n))$,
(ii) A_5 is regular, and
(iii) $A_4 A_5 = A$.

From these facts and Lemma 2.1, (2) follows.

Proof of (3): Let

$$\begin{aligned} A_6 = \{&bin(1)\#bin(2)\#\ldots\#bin(n)\# \\ &wcw_{11}cw_{12}c\ldots cw_{1r_1}ccw_{21}cw_{22}c\ldots cw_{2r_2} \\ &\in \{0,1,c,\#\}^+ \mid n \ge 2 \;\&\; w \in \{0,1\}^{\lceil \log n \rceil} \;\& \\ &\forall k\,(1 \le k \le 2)[r_k \ge 1 \;\& \\ &\forall l\,(1 \le l \le r_k)[w_{kl} \in \{0,1\}^+]]\}. \end{aligned}$$

It is obvious that $A_4 \cup A_5 \in$ *strong*-2NTM$^*(L(n))$. We can observe that $A_6 \in$ *strong*-2DTM$(L(n))$. Suppose that $(A_4 \cup A_5)^*$ is in *weak*-2NTM$^*(L(n))$. Then, $(A_4 \cup A_5)^* \cap A_6$ is also in *weak*-2NTM$^*(L(n))$. This is a contradiction, since $(A_4 \cup A_5)^* \cap A_6 = A$. □

In order to obtain our result that m-2UTM$^*(L(n))$ is not closed under the language operations mentioned above, we prepare the following lemma which is shown in Ref. 7).

Lemma 2.2. Let

$$T = \{bin(1)\#bin(2)\#\ldots\#bin(n)\# cw_1cw_2c\ldots cw_rccu_1cu_2c\ldots cu_{r'} \in \{0,1,c,\#\}^+ \mid n \geq 2 \;\&\; r, r' \geq 1 \;\&\; \forall k\,(1 \leq k \leq r)\,[w_k \in \{0,1\}^{\lceil \log n \rceil}] \;\&\; \forall l\,(1 \leq l \leq r')\,[u_l \in \{0,1\}^{\lceil \log n \rceil}] \;\&\; \exists i\,(1 \leq i \leq r')\,[\forall j\,(1 \leq j \leq r)[u_i \neq w_j]]\}.$$

Then, $T \notin$ *weak*-2UTM$^*(L(n))$.

Theorem 2.4. m-2UTM$^*(L(n))$ is not closed under
(1) length-preserving homomorphism,
(2) concatenation with regular sets, and
(3) Kleene closure.

Proof of (1): Let

$$T_1 = \{bin(1)\#bin(2)\#\ldots\#bin(n)\# cw_1cw_2c\ldots cw_rcc_1u_1c_2u_2c_3\ldots c_{r'}u_{r'} \in \{0,1,c,d,\#\}^+ \mid n \geq 2 \;\&\; r, r' \geq 1 \;\&\; \forall k\,(1 \leq k \leq r)[w_k \in \{0,1\}^{\lceil \log n \rceil}] \;\&\; \forall l\,(1 \leq l \leq r')\,[u_l \in \{0,1\}^{\lceil \log n \rceil}] \;\&\; \exists i\,(1 \leq i \leq r')\,[c_i = d \;\&\; \forall j\,(1 \leq j \leq r)[u_i \neq w_j] \;\&\; \forall p\,(1 \leq p \leq r, i \neq p)[c_p = c]]\},$$

and let h be the homomorphism defined in the proof of Theorem 2.3. Then, we can show that $h(T_1) = T$. So, from this fact and Lemma 2.2, (1) follows.

Proof of (2): Let

$$T_2 = \{bin(1)\#bin(2)\#\ldots\#bin(n)\# cw_1cw_2c\ldots cw_rccu_1cu_2c\ldots cu_{r'} \in \{0,1,c,\#\}^+ \mid n \geq 2 \;\&\; r, r' \geq 1 \;\&\; \forall k\,(1 \leq k \leq r)\,[w_k \in \{0,1\}^{\lceil \log n \rceil}] \;\&\; \forall l\,(1 \leq l \leq r')\,[u_l \in \{0,1\}^{\lceil \log n \rceil}] \;\&\; \forall j\,(1 \leq j \leq r)[u_{r'} \neq w_j]]\}$$

and let A_5 be the language defined in the proof of Theorem 2.3. Then, it is clear that
(i) $T_2 \in$ *strong*- 2UTM$^*(L(n))$ and
(ii) $T_2A_5 = T$.
(2) follows from these facts and Lemma 2.2.

Proof of (3): It is trivial that $T_2 \cup A_5$ is in *strong*-2UTM$^*(L(n))$. Let

$$T_3 = \{bin(1)\#bin(2)\#\ldots\#bin(n)\# cw_1cw_2c\ldots cw_rccu_1cu_2c\ldots cu_{r'} \in \{0,1,c,\#\}^+ \mid n \geq 2 \;\&\; r, r' \geq 1 \;\&\; \forall k\,(1 \leq k \leq r)\,[w_k \in \{0,1\}^{\lceil \log n \rceil}] \; \forall l\,(1 \leq l \leq r')\,[u_l \in \{0,1\}^+]\}.$$

Then, it is clearly shown that $T_3 \in$ *strong*-2UTM$^*(L(n))$ and $(T_2 \cup A_5)^* \cap T_3 = T$. From these facts and Lemma 2.2, (3) follows. □

3. Concluding Remarks

We investigate closure property of sublogarithmic space-bounded 1-inkdot 2-way alternating Turing machines with only existential (universal) states.

Our main result is that m-2NTM$^*(L(n))$ and m-2UTM$^*(L(n))$ are not closed under intersection and union, respectively, and both of them are not closed under complementation, length-preserving homomorphism, concatenation with regular sets, and Kleene closure.

For determinism, it is shown in Ref. 2) that

"m-2DTM$^*(L(n)) = m$-2DTM$(L(n))$"

(For closure properties of m-2DTM$(L(n))$ (thus, m-2DTM$^*(L(n))$), see Refs. 3), 4)).

Unfortunately, whether m-2ATM$^*(L(n))$ is closed under the operations discussed in this paper except for union and intersection is an open problem.

Inoue et al. introduced in Ref. 8) a multi-inkdot Turing machines as an extension of the 1-inkdot Turing machine. Some of the results obtained here will be able to be extended to the multi-inkdot version. We will give them in a forthcoming paper.

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