A STUDY ON A VEHICLE ROUTING PROBLEM FOR REUSE SYSTEMS OF TRANSPORT PACKAGES

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Abstract

This paper aims to propose a heuristics on a vehicle routing problem for the reuse systems of transport packages. With the operations of reuse and/or recycling systems, effective systems for reverse logistics which disposed products are collected from customers are needed. In this paper, a vehicle routing problem for the reuse system of transport package is considered, which disposed products are transported by empty capacity of many vehicles which move toward their destinations or return to their originations.This paper proposes a heuristics using some limited information with respect to vehicle routines, and clarifies the effectiveness of our proposed heuristics from some numerical examples.

Key Words: Reverse logistics, Reuse system, Transport package, Vehicle routing problem

1. Introduction

Recently, on the viewing point of environmental protection, many manufacturers construct reuse and/or recycling systems for disposed products. With the operations of reuse and/or recycling systems, effective systems for reverse logistics which disposed products are collected from customers are needed. In this paper, a vehicle routing problem for the reuse system of transport package is considered, which disposed transport package such as corrugated cardboard, flexible container bag, etc. are transported by empty capacity of many vehicles which move toward their destinations or return to their originations.

This paper investigates the following subjects in order to propose a heuristics on a vehicle routing problem for the reuse systems of transport packages.

(1) Review on the studies of the reverse logistics.

(2) Formulate a vehicle routing model for a mathematical programming problem, which maximizes the total transportation quantity of disposed transport package.

(3) Propose a heuristics using some limited information with respect to vehicle routines.

(4) Clarify the effectiveness of our proposed heuristics from some numerical examples.

2. A review on studies of reverse logistics

The study of reverse logistics is a remarkable field, and many researcheres have studied the reuse and/or recycling systems which are operated of the specified product, of the specified industry, or in the specified country. Fleischmann et al.[1] point out some characteristics of reverse logistics as follows.

(1) The activities of reverse logistics are collection, inspection/separation, re-processing, disposal, and re-distribution, and they construct three phases, that is, collection , product recovery and re-distribution phase.

(2) In 'closed-looped' case, interaction between collection and re-distribution activities may add complexity to the network design.

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(3) Availability of used products trigger the sequence of activities.

(4) Time restriction tends to be weaker for collection.

(5) Timing and quantity of used products coming free are determined by the former user.

And the classification of reverse logistics networks are represented as follows.

(1) Bulk recycling network.

(2) Assembly product remanufacturing network.

(3) Re-use item network.

In Fleischmann et al.[2], the study of the reverse logistic is classified into the study of the location of the facilities and of the production and inventory control considering the collected disposal products. Barros et al.[3] report on a case study addressing the design of a logistics network for recycling sand coming free from processing construction waste in The Netherlands. To determine the optimal number, capacity, and location of the depots and cleaning facilities, the authors propose a multi-level capacitated facility location model which is solved approximately via iterative rounding of LP-relaxations strengthtened by valid inequalities. Louwers et al.[4] consider the recycling network for carpet waste. To determine the appropriate locations and capacities for regional recovery centers, the authors propose a continuous location model. Using a linear approximation of the share of fixed costs per volume processed. The resulting nonlinear model is solved using standard software. Krikke et al.[5] present a mixed integer linear program model for a multi-echelon reverse logistic network for durable consumer products. This model is applied in a case study carried out a copier manufacturer in The Netherlans. Kroon and Vrijens [6] consider the design of a logistcs system for reusable transport packages, that is, collapsible plastic containers. A closed-loop deposit based system is considerd.

Fleischmann et al.[7] and Fleischmann and Kuik [8] propose the the method of inventory control, and analyze the effects of the fluctuation of the quantity of collected products. Minner [9] proposes the nonlinear programming model for minimizing the holding cost of buffer stock for the supply chain model considering the recycling of disposal prodcts and the reuse of by-products.

In this paper, we consider the vehicle routing problem for the reuse systems of the transport packages. The disposed transport package such as corrugated cardboard, flexible container bag, etc. are transported by empty capacity of many vehicles which move toward their destinations or return to their originations. The characteristcs of the reuse system of transport packages in this paper are pointed out as follows.

(1) Time restriction of the delivery of the transport packages is weaker.

(2) Empty capacity of the transport vehicles is used effectively.

(3) Transport packages are transported as many as possible with feasible transport vehicles.

(4) Plural transport vehicles can transport the transport packages among the same collected and/or reuse bases(CRB).

Considering the above characteristics of the reuse systems of the transport packages, we formulate a vehicle routing model for the reuse systems of the transport packages.

3. Formulating a vehicle routing model for reuse system

3.1 Assumption

In this paper, we consider a vehicle routing problem for reuse system which satisfies the following conditions.

(1) Volume of collected transport packages is given.

(2) Transportation cost of each vehicle is varied by transportation quantity and transported distance.

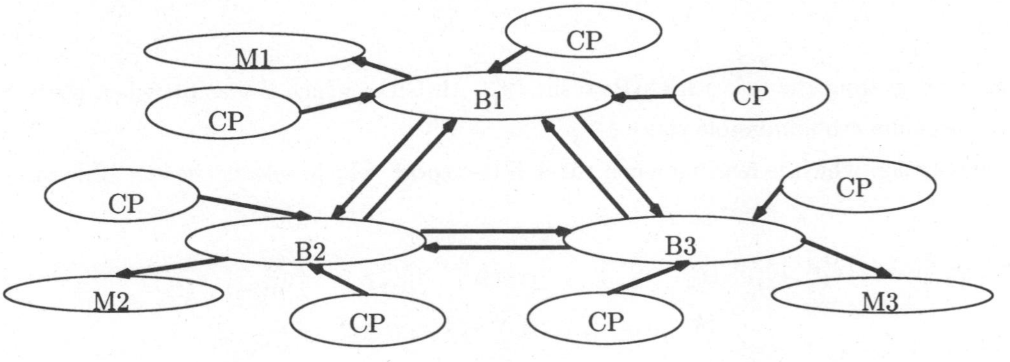
(3) Remaining of the transport packages which is not transported is permitted.

(4) Departure and arrival time of the vehicle are kept.

(5) Transportation of plural vehicles among same CRBs is permitted.

(6) Travel time among the CRBs is recorded in the database, and all admissible route for each vehicle can be extracted.

Figure 1 shows the schematic diagram of the reuse system of the transport packages. We consider the vehicle routing problem among the CRBs.



B1,B2,B3: Colleced and/or Reuse Base, CP: Collected Point
M1,M2,M3: Manufacturer

Figure 1: The schematic diagram of the reuse system of the transport packages

3.2 Notations

Let us define notations as follows.

N: the number of CRBs.

t_{ij}: the travel time from the ith CRB to the jth CRB.

D_{ij}: the collected quantity of transport package at the ith CRB which is transported to the jth CRB.

W_{ij}: the coefficient of weight which represents the priority of transportation from the ith CRB to the jth CRB.

K : the number of transport vehicles.

C^k : the capacity of the kth transport vehicle.

T^k : the permissible time of the kth transport vehicle.

v_S^k : the starting position of the kth transport vehicle.

v_E^k : the destination of the kth transport vehicle.

$t_i^{k,S}$: the travel time which the kth transport vehicle moves from starting position to the ith CRB.

$t_j^{k,E}$: the travel time which the kth transport vehicle moves from the jth CRB to the destination.

$\bar{M}_r^k$: the number of the passed CRBs which the kth transport vehicle moves along the rth route.

$v_i^{k,r}$: the CRB which the kth transport vehicle visits the ith order along the rth route.

R^k : the number of admissible routes of the kth transport vehicle.

g_i : the service time at the the ith CRB.

From the information of the kth transport vehicle, admissible routes are extracted from the database of travel time among the CRBs by inequality (1).

$$t_{v_1^{k,r}}^{k,S} + \sum_{i=1}^{\bar{M}_r^k-1} t_{v_i^{k,r} v_{i+1}^{k,r}} + t_{v_{\bar{M}_r^k}^{k,r}}^{k,E} + \sum_{i=1}^{\bar{M}_r^k} g_i \leq T^k \qquad (k=1,\cdots,K, r=1,\cdots,R^k) \tag{1}$$

Extracting admissible routes from the database can find all admissible route, however, in this paper, it assumes to extract admissible routes by upper limited number [10] .

$\langle v_S^k, v_1^{k,r}, \cdots, v_{\bar{M}_r^k}^{k,r}, v_E^k \rangle$: the rth admissible route which is passed through the CRBs of the kth transport vehicle.

Q^k : the set of the admissible routes of the kth transport vehicle as follows.

$$Q^k = \{\langle v_S^k, v_1^{k,1}, \cdots, v_{\bar{M}_1^k}^{k,1}, v_E^k \rangle, \langle v_S^k, v_1^{k,2}, \quad \cdots \quad, v_{\bar{M}_2^k}^{k,2}, v_E^k \rangle, \cdots, \langle v_S^k, v_1^{k,R^k}, \cdots, v_{\bar{M}_{R^k}^k}^{k,R^k}, v_E^k \rangle\}$$
$$(k=1,\cdots,K) \tag{2}$$

$(v_i^{k,r}, v_j^{k,r})$: the transportation from the $(v_i^{k,r})$th CRB to the $(v_j^{k,r})$th CRB, which is feasible when the kth transport vehicle selects the rth admissible route .

S_r^k : the set of the transpotation which is feasible when the kth transport vehicle selects the rth admissible route as follows.

$$S_r^k = \{(v_1^{k,r}, v_2^{k,r}), \cdots, (v_1^{k,r}, v_{\bar{M}_r^k}^{k,r}), (v_2^{k,r}, v_3^{k,r}) \quad , \quad \cdots, (v_2^{k,r}, v_{\bar{M}_r^k}^{k,r}), \cdots, (v_{\bar{M}_r^k-1}^{k,r}, v_{\bar{M}_r^k}^{k,r})\}$$
$$(k=1,\cdots \quad , \quad K, r=1,\cdots,R^k) \tag{3}$$

$\delta_{ij}^{k,r}$: the binary coefficient which represents the transportation from the ith CRB to the jth CRB, which is feasible the kth transport vehicle selects the rth admissible route as follows.

$$\delta_{ij}^{k,r} = \begin{cases} 1 & (i,j) \in S_r^k \\ 0 & (i,j) \notin S_r^k \end{cases} \tag{4}$$

$P_{ij}^{k,r}$: the transportation quantity from the ith CRB to the jth CRB, which the kth transport vehicle selects the rth admissible route .

X_r^k : the binary variable, which represents $X_r^k = 1$ when which the kth transport vehicle selects the rth admissible route, $X_r^k = 0$ otherwise.

Using the above notations, we formulate the transportation planning model among the CRBs.

3.3 Formulation

To transport the collected transport packages as many as possible, we formulate the transportation planning model which selects the routes of all transport vehicle that the total transportation quantity is maximized.

(1) Objective function

$$Z = \sum_{k=1}^{K} \sum_{r=1}^{R^k} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} W_{ij} \cdot \delta_{ij}^{k,r} \cdot P_{ij}^{k,r} \longrightarrow Max \tag{5}$$

Objective function(5) represents to maxmize the total transportation quantity considering the priority of the transportation among the CRBs.

(2) Constraints with respect to the collected quantity of transport package

$$\sum_{k=1}^{K}\sum_{r=1}^{R^k}\delta_{ij}^{k,r}\cdot P_{ij}^{k,r} \leq D_{ij} \qquad (i=1,\cdots,N, j=1,\cdots,N, i\neq j) \tag{6}$$

Inequality(6) repersents that total transportation quanatity of the transport vehicles must be equal or less than the collected quantity of transport package.

(3) Constraints with respect to the capacity of the transport vehicle

$$\sum_{i=1}^{j}\sum_{m=j+1}^{\bar{M}_r^k}\delta_{v_i^{k,r}v_m^{k,r}}^{k,r}\cdot P_{v_i^{k,r}v_m^{k,r}}^{k,r} \leq C^k\cdot X_r^k$$
$$(k=1,\cdots,K, r=1,\cdots,R^k, j=1,\cdots,\bar{M}_r^k-1 \quad , \quad (v_i^{k,r}v_m^{k,r})\in S_r^k) \tag{7}$$

Inequality(7) represents that the carrying quantity of the transport vehicle at the CRBs is eqaul or less than the capacity of the transport vehicle.

(4) Constraints with respect to the admissible route of the transport vehicle

$$\delta_{ij}^{k,r}\cdot P_{ij}^{k,r} \leq \; V \; \cdot X_r^k \qquad (k=1,\cdots,K, r=1,\cdots,R^k, (i,j)\in S_r^k) \tag{8}$$

$$\sum_{r=1}^{R^k} X_r^k = 1 \qquad (k=1,\cdots,K) \tag{9}$$

$$X_r^k \in \{0,1\} \qquad (k=1,\cdots,K, r=1,\cdots,R^k) \tag{10}$$

Equation and inequality (8)-(10) represent that only one admissible route must be selected among the set of the admissible routes, where, V is a large positive number.

(5) Non-negative constraint of the transportation quantity

$$P_{ij}^{k,r} \geq 0 \qquad (k=1,\cdots,K, r=1,\cdots,R^k, (i,j)\in S_r^k) \tag{11}$$

We call the mathematical Programming model which maximizes objective function (5) subjects to eqnations and inequalities (6)-(11), the transportation planning model among the CRBs.

4.Heuristics

In order to solve the transportation planning model among the CRBs, we propose a heuristic method using the limited routes of the transport vehicle extracted from the database of the travel time among the CRBs, because it requires much calculation time to calculate an exact solution using all route of the transport vehicle, and using Lagrangian relaxation as subproblems, solving the Lagrangian dual with dual simplex method [11],[12].

By relaxing the inequality (6) using Lagrangian multiplier $u=(u_{ij})$, the Lagrangian relaxation problem is separated into the subproblems of each transport vehicle as follows.

$$Z_D(u) \;=\; \sum_{k=1}^{K} Z_D^k(u) - \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} u_{ij}\cdot D_{ij} \longrightarrow Max \tag{12}$$

$$Z_D^k(u) \;=\; \sum_{r=1}^{R^k}\sum_{i=1}^{N}\sum_{j=1,i\neq j}^{N}(W_{ij}-u_{ij})\cdot\delta_{ij}^{k,r}\cdot P_{ij}^{k,r} \longrightarrow Max \tag{13}$$

$$s.t.$$

$$\sum_{i=1}^{j}\sum_{m=j+1}^{\bar{M}_r^k} \delta^{k,r}_{v_i^{k,r}v_m^{k,r}} \cdot P^{k,r}_{v_i^{k,r}v_m^{k,r}} \leq C^k \cdot X_r^k \tag{14}$$

$$(r = 1, \cdots, R^k, j = 1, \cdots, \bar{M}_r^k - 1, (v_i^{k,r} v_m^{k,r}) \in S_r^k) \tag{15}$$

$$\delta_{ij}^{k,r} \cdot P_{ij}^{k,r} \leq V \cdot X_r^k \qquad (r = 1, \cdots, R^k, (i,j) \in S_r^k) \tag{16}$$

$$\sum_{r=1}^{R^k} X_r^k = 1 \tag{17}$$

$$X_r^k \in \{0,1\} \qquad (r = 1, \cdots, R^k) \tag{18}$$

$$P_{ij}^{k,r} \geq 0 \qquad (r = 1, \cdots, R^k, (i,j) \in S_r^k) \tag{19}$$

As separated Lagrangian subproblems have integrality property, subproblems can be solved easily by linear programming method. However, we solve the separated Lagrangian subproblem by another method. The separated Lagrangian subproblem are separated again into each admissible route, called re-separated subproblem. Re-separated subproblems are simple LP problems subject to the only constraint of the capacity of transport vehicles. By sorting the admissible routes of each transport vehicle according to the large value of objective function of initial re-separeted subproblems when Lagrangian multipliers equal zero, all re-separated subproblm do not necessarily solve because the upper bounded value of each re-separated subproblem is already calculated. Lagrangian dual is soloved by dual simplex method, adding the constraints obtained from the solutions of Lagrangian subproblems.

$$Z_D = \sum_{k=1}^{K} w_k - \sum_{i=1}^{N}\sum_{j=1,i\neq j}^{N} u_{ij} \cdot D_{ij} \longrightarrow Min \tag{20}$$

$$s.t.$$

$$w_k \geq \sum_{i=1}^{N}\sum_{j=1,i\neq j}^{N} (W_{ij} - u_{ij}) \cdot \delta_{ij}^{k,r} \cdot \bar{P}_{ij,l}^{k,r}$$

$$(k = 1, \cdots, K, r = 1, \cdots, R^k, l = 1, \cdots, L) \tag{21}$$

where, $\bar{P}_{ij,l}^{k,r}$ is a solution of subproblem at the lth iteration and L is the number of iterations.

From the simplex multipliers of the Lagrangian dual, the primal solution of another form of Lagrangian dual which is based on Dantzig-Wolfe Dicomposition is obtained. Feasible solution and/or branching variable is found from the primal solution, and using branching and bound procedure, best feasible solution of the transportation planning model is found.

5. Numerical Examples

In order to clarify the effectiveness of the proposed heuristics, some numerical examples are shown under the following conditions.

(1) Number of CRBs

$$N = 9, 12 \tag{22}$$

Table 1 shows the travel time among the CRBs.

(2) Number of transport vehicles

$$K = 10, 15 \tag{23}$$

Table 1: Travel Time among the CRBs

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
C1	0.0	2.5	0.9	4.0	3.1	6.6	6.5	3.8	7.9	2.9	5.5	7.7
C2	2.5	0.0	3.0	6.0	5.4	8.6	8.5	6.3	8.9	4.7	8.0	10.0
C3	0.9	3.0	0.0	3.1	3.2	5.8	5.7	3.6	7.1	2.0	5.2	7.0
C4	4.0	6.0	3.1	0.0	4.2	2.7	2.6	3.7	5.0	1.3	4.0	4.2
C5	3.1	5.4	3.2	4.2	0.0	6.5	6.1	1.2	9.1	4.0	3.0	6.4
C6	6.6	8.6	5.8	2.7	6.5	0.0	0.5	5.6	3.9	3.8	5.0	2.7
C7	6.5	8.5	5.7	2.6	6.1	0.5	0.0	5.2	4.5	3.8	4.6	2.2
C8	3.8	6.3	3.6	3.7	1.2	5.6	5.2	0.0	8.6	3.9	1.8	5.3
C9	7.9	8.9	7.1	5.0	9.1	3.9	4.5	8.6	0.0	5.3	8.6	6.5
C10	2.9	4.7	2.0	1.3	4.0	3.8	3.8	3.9	5.3	0.0	4.8	5.5
C11	5.5	8.0	5.2	4.0	3.0	5.0	4.6	1.8	8.6	4.8	0.0	3.9
C12	7.7	10.0	7.0	4.2	6.4	2.7	2.2	5.3	6.5	5.5	3.9	0.0

Table 2: Collected quantity of the transport packages

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
C1	0	500	300	300	200	200	200	100	400	300	100	100
C2	500	0	400	300	500	200	200	500	100	300	400	200
C3	300	300	0	200	400	100	400	300	300	100	400	400
C4	200	100	400	0	400	100	500	300	300	500	100	400
C5	100	200	100	400	0	500	100	300	200	300	300	500
C6	500	200	400	200	200	0	200	200	100	400	300	200
C7	200	500	100	400	300	400	0	200	200	500	100	300
C8	500	100	100	400	400	300	200	0	400	200	300	200
C9	100	200	500	500	100	100	400	200	0	500	400	300
C10	100	200	100	200	400	300	100	400	500	0	100	200
C11	200	300	300	400	200	400	300	300	500	200	0	400
C12	100	200	500	400	400	300	300	400	200	500	500	0

Table 3: Permissible Time and number of admissible route of the transport vehicles

Transport Vehicle	Permissible Time	Number of Admissible routes
M1	13.9	7
M2	12.4	1
M3	12.5	6
M4	10.5	5
M5	19.6	17
M6	7.8	3
M7	15.0	16
M8	8.9	1
M9	9.8	6
M10	11.6	20
M11	15.6	18
M12	20.7	40
M13	14.8	8
M14	21.7	48
M15	6..8	1

(3) Capacity of transport vehicles

$$C^k = 300 \qquad (k = 1, \cdots, K)$$

(4) Collected quantity of the transport packages

Table 2 shows the collected quantity of the transport packages which transport from the ith CRB to jth CRB.

(5) Permissible time and number of admissible route of the transport vehicles

$$T^k_{max} = SD_k * (1.0 + 0.3) \tag{24}$$

where, SD_k is the travel time of the kth transport vehicle from starting postion to destination. Table 3 shows the permissible time and the number of admissible route of the transport vehicles.

Table 4: The result of the Numerical Example ($N = 9, K = 10$)

Transport Vehicle	Selected Route	Transportation Quantity							
M1	$2 \to 3 \to 5$	$2 \to 3$	300	$3 \to 5$	300				
M2	$1 \to 3$	$1 \to 3$	300						
M3	$3 \to 2$	$3 \to 2$	300						
M4	$7 \to 8$	$7 \to 8$	200						
M5	$7 \to 6 \to 9$	$6 \to 9$	100	$7 \to 6$	100	$7 \to 9$	200		
M6	$6 \to 7$	$6 \to 7$	200						
M7	$2 \to 1 \to 5 \to 8$	$1 \to 5$	200	$2 \to 1$	200	$2 \to 5$	100	$5 \to 8$	300
M8	$7 \to 6$	$7 \to 6$	300						
M9	$3 \to 8$	$3 \to 8$	300						
M10	$3 \to 4$	$3 \to 4$	200	$4 \to 7$	300				
Total transportation quantity								3900	

Table 4 shows the result of numerical example. This table shows that each transport vehicle is assigned the transportation among the CRBs. Table 5 shows the results of the numerical examples varying number of the CRBs, number of the transport packages and upper limit of admissible route. In this table, when the number of CRB is 9, we solved the transportation planning model used the collected quantity of the transport package from 1st CRB(C1) to 9th CRB(C9), and when the number of transport vehicles is 10, used the data of transport vehicles from M1 to M10. Form this table, when the number of the CRBs and number of transport vehicles increase, processing times increases. When maximum number of the extracted routes from the database decreases, total transportation quantity does not decrease so much. And, it seems that processing time decreases when the maximum number of the extracted routes decreases. These results show that the proposed heuristic method provides a practical transportation plan using limited number of extracted routes.

6. Conclusion

This paper proposes a heuristics on a vehicle routing problem for reuse systems of transport packages. The followings are clarified.

(1) Formulate a vehicle routing model for a mathematical programming problem, which maximizes the total transportation quantity of disposed products.

Table 5: The results of the numerical examples varying N, K and upper limit of R^k

Number of CRBs	Number of Transport Vehicles	Upper Limit of Admissible Routes	Objective Function	CPU Time (sec)
9	10	5	3900	0.3
9	10	10	3900	0.89
9	10	15	3900	0.92
9	10	all	3900	1.02
9	15	5	6400	3.52
9	15	10	6600	6.96
9	15	15	6600	9.21
9	15	all	6600	11.05
12	10	5	4400	2.35
12	10	10	4400	3.08
12	10	15	4400	7.68
12	10	all	4400	8.27
12	15	5	8100	54.6
12	15	10	8100	103.7
12	15	15	8300	152.1
12	15	all	8400	253.6

(2) Propose a heuristics using some limited information with respect to vehicle routines.

(3) Clarify the effectiveness of our proposed heuristics from some numerical examples.

Further studies are necessary to examine a vehicle routing problem considering the transportation using the intermidiate CRBs.

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