

Studies on the Added Mass and Added Moment of Inertia of Propellers-II*

Effects of Pitch Angle on Controllable Pitch Propellers

By

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In the case a controllable pitch propeller, the values of the added mass and added moment of inertia are changed by varying the pitch angle during operation. It can be noted that the value of the added polar moment of inertia is greatly influenced by the pitch angle. However, there are few papers about the effects of pitch angle on the added mass and added moment of inertia. Therefore, in this paper, in order to evaluate the pitch angle, which has a great effect on the added polar moment of inertia of the propeller, the experimental studies were carried out using a model propeller which could change only the pitch angle. As a result, the following can be understood: 1. It is necessary to consider the effects of pitch angle when designing a propeller shaft on which a controllable pitch propeller is mounted. 2. The two-dimensional oscillating wing theory, corrected by the coefficient of the threedimensional correction J_T , is very effective for calculating the added polar moment of inertia of a controllable pitch propeller. 3. In a state of resonance which is under torsional vibration, the value of the unsteady characteristic can be determined by the order of the torsional vibration, the radial position of the propeller and the chord length of a blade. 4. The value of the unsteady characteristic can be represented by the value at the radial position of a propeller $0.7R$.

1. Introduction

In the previous paper¹⁾, when the principal specifications of a propeller were given, it was shown that the added polar moment of inertia of a fixed propeller could be accurately calculated by applying the three-dimensional correction²⁾ to the value which was obtained through the potential theory³⁾ or the two-dimensional oscillating wing theory.^{4,5)}

But, in the case of a controllable pitch propeller the values of the added mass and added moment of inertia are changed by varying the pitch angle during operation. It can be noted that the value of the added polar moment of inertia is greatly influenced by the pitch angle. However, there are few papers^{3,6)} about the effects of pitch angle on added mass and added moment of inertia. Also, in these papers there are some

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faults: the experiments are narrowly limited in the range of pitch angle. And other facts influence their results; differences of the blade form and blade area ratio etc, because of the use of series model propellers in the tests. These papers are insufficient for the evaluation of the added polar moment of inertia of a propeller such as a controllable pitch propeller, for which only pitch angle can be varied.

Therefore, in this paper, in order to evaluate the pitch angle, which has a great effect on the added polar moment of inertia of a propeller, the experimental studies were carried out using a model propeller which could change only the pitch angle.

2. Added Polar Moment of Inertia

2.1 Nomenclature

- V : velocity of steady flow (m/s)
 Z_a : distance between datum position and centre of wing (m)
 θ : angle of attack to steady flow V (rad)
 B : chord length of wing or propeller blade (m)
 ω : circular frequency of wing (rad/s)
 lift in vertical direction to steady flow V (N)
 ρ : density (kg/m³)
 $C(\nu)$: THEODORSEN function
 ν : reduced frequency
 J_0, J_1 : BESSEL function of the first kind of zero and one order
 Y_0, Y_1 : BESSEL function of the second kind of zero and one order
 $\beta(r)$: angle in tangential direction of propeller to steady flow V at radial position r of propeller (rad)
 Z : number of blades
 I_{pw} : added polar moment of inertia of propeller

2.2 Fundamental Equation

The two-dimensional oscillating wing theory, which can explain the physical mean of the phenomena, is the most effective method of evaluating the added mass and moment of inertia of a propeller⁷⁾.

Therefore, this theory may be applied to the added polar moment of inertia of a propeller.

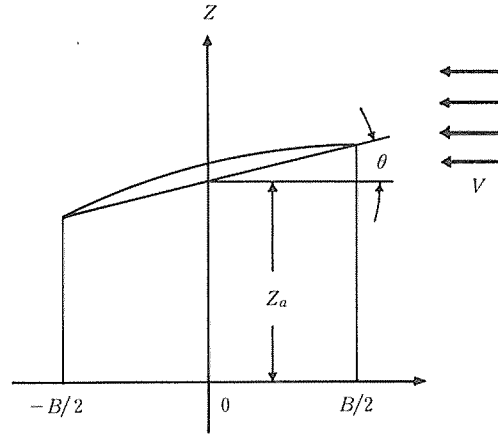


Fig. 1. Two-dimensional thin wing.

As shown in Fig. 1, for a thin two-dimensional wing which has translational motion in the vertical direction to the steady flow, and also has rotational motion round the centre of the wing, the unsteady lift of the wing is shown in equation⁸⁾ (1).

$$L(t) = \pi\rho\left(\frac{B}{2}\right)^2(-\dot{Z}_a + V\dot{\theta}) + \pi\rho VBC(\nu)\left(-\dot{Z}_a + \frac{B}{4}\dot{\theta} + V\theta\right) \quad (1)$$

where $C(\nu)$, the complex function in which reduced frequency ν is parameter, namely THEODORSEN function, is given by equation (2).

$$C(\nu) = \left. \begin{aligned} &F(\nu) + iG(\nu) \\ &= \frac{J_1(\nu) - iY_1(\nu)}{J_1(\nu) + Y_0(\nu) + i\{J_0(\nu) - Y_1(\nu)\}} \end{aligned} \right\} \quad (2)$$

$$\nu = \frac{B\omega}{2V}$$

when applying equation (1) to the blade of a propeller, assuming that these motions are under the harmonic vibration of frequency ω , it can be shown that

$$Z_a = \bar{Z}_a e^{i\omega t}, \quad \theta = \bar{\theta} e^{i\omega t} \quad (3)$$

If equation (1) is rearranged using equation (3), the following equation is obtained,

$$\begin{aligned}
L(t) = & -\frac{1}{4}\pi\rho B^2\left\{1+\frac{2G(\nu)}{\nu}\right\}\ddot{Z}_a - \pi\rho BVF(\nu)\dot{Z}_a \\
& + \frac{1}{6}\pi\rho B^3\left\{\frac{G(\nu)}{\nu}\right\}\dot{\theta} + \frac{1}{4}\pi\rho B^2 V\left\{1+F(\nu)\right. \\
& \left. + \frac{2G(\nu)}{\nu}\right\}\dot{\theta} + \pi\rho BV^2 F(\nu)\theta
\end{aligned} \quad (4)$$

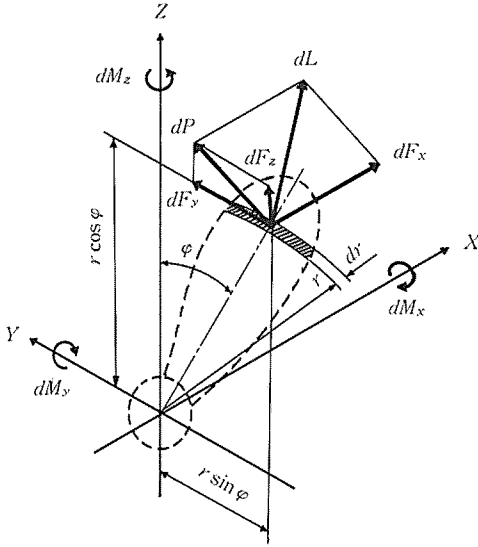


Fig. 2. Components illustrating the hydrodynamic forces exerted on a propeller blade.

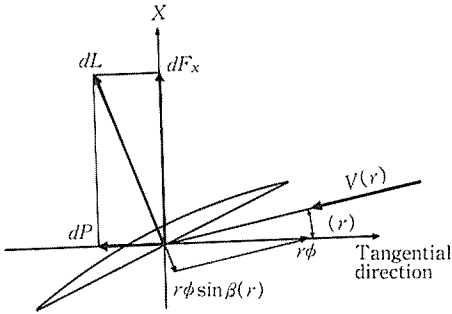


Fig. 3. Translational motion of a propeller blade and hydrodynamic forces under torsional vibration.

Next, we apply this theory to a propeller which is under torsional vibration $\phi = \bar{\phi}e^{i\omega t}$. The coordinates of the propeller motion are shown in Fig. 2. As shown in Fig. 3, the element of a blade between the radius r and $r+dr$ has the translational motion of $-r\phi\sin\beta(r)$ in the vertical direction to the steady flow V under the torsional vibration. The lift which is produced by this translational motion becomes the following equation,

$$\begin{aligned}
dL = & \frac{1}{4}\pi\rho rB(r)^2\sin\beta(r)\left\{1+\frac{2G(\nu_r)}{\nu_r}\right\}dr\ddot{\phi} \\
& + \pi\rho rB(r)V(r)F(\nu_r)\sin\beta(r)dr\dot{\phi}
\end{aligned} \quad (5)$$

From Fig. 2 and Fig. 3, the force and moment which are produced by the lift can be written as follows:

$$\left. \begin{aligned}
dF_x &= dL\cos\beta(r) \\
dF_y &= dL\sin\beta(r)\cos\phi \\
dF_z &= dL\sin\beta(r)\sin\phi \\
dM_x &= -rdL\sin\beta(r) \\
dM_y &= rdL\cos\beta(r)\cos\phi \\
dM_z &= rdL\cos\beta(r)\sin\phi
\end{aligned} \right\} \quad (6)$$

where the sum of the number of blades Z on these six components become

$$\sum_{n=1}^Z \sin\phi = 0, \quad \sum_{n=1}^Z \cos\phi = 0 \quad (7)$$

Namely, the sum of the hydrodynamic forces which are produced by torsional vibration are only the component of thrust dF_x and the component of torque dM_x round the X -axis. The following equations are derived from the equation of motion of a wing.

$$\begin{aligned}
M_x &= -I_{pw}\ddot{\phi} - C_{pw}\dot{\phi} \\
F_x &= -I_{rw}\ddot{\phi} - C_{rw}\dot{\phi}
\end{aligned} \quad (8)$$

Consequently, multiplying dM_x by the number of blades Z and integrating it with respect to r , the added polar moment of inertia of the propeller under torsional vibration can be obtained as follows,

$$I_{pw} = \frac{\pi\rho Z}{4} \int_{r_0}^R r^2 \{B(r)\sin\beta(r)\}^2 \left\{1 + \frac{2G(\nu_r)}{\nu_r}\right\} dr \quad (9)$$

3. Experiment

The Experimental apparatus is shown in Fig. 4. An explanation of this apparatus has been omitted because the details have already been mentioned in the previous paper.

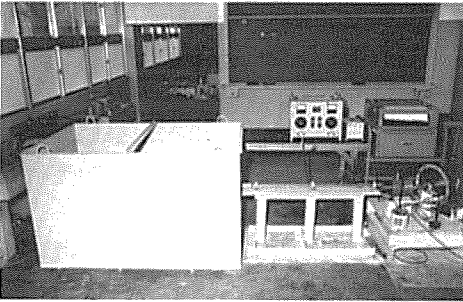


Fig. 4. Photograph of experimental apparatus.

3.1 Model Propeller

For investigating the effects of pitch angle on the added polar moment of inertia, a propeller which can change only the pitch angle, such as a controllable pitch propeller, is better than the series model propellers which have various pitch angles.

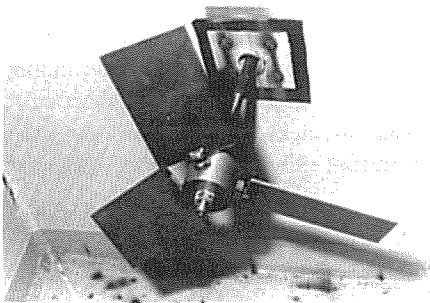


Fig. 5. Model propeller.

Therefore, as shown in Fig. 5, a propeller which can change only pitch angle was produced. The blade can be fixed on the boss using a nut to change the pitch angle easily. A steel plate (thickness of 3 mm) is used for the blade of the propeller because the blade of a propeller is essentially a flat blade, and also the effects of the thickness of a blade and sectional form are negligible under torsional vibration.^{2,3)} The propeller's principal specifications are shown in Table 1.

Table 1. Principal specifications of the model propeller

Number of blades	3
Diameter (cm)	37.7
Boss ratio	0.138
Exp. area ratio	0.323
Direction of turn	Right
Mass (kg)	1.550
Aspect ratio	1.875

3.2 Experimental Method

The value of the added polar moment of inertia of a propeller can be obtained from the difference between the resonance frequencies of the torsional vibration system in the air and in water. The method of measurement for the accurate resonance frequency of the torsional vibration system is the same as that used in the first paper of this study.

In this experiment, it is necessary to measure the pitch angle accurately. The method of measurement is shown in Fig. 6. The pitch angle is obtained from the difference between h_1 and h_2 because the chord length of a blade is constant. The experimental range of the pitch angle is from 0° to 45° rising every 5° . The practical range of the pitch angle for a controllable pitch propeller is generally between -20° and $+20^\circ$. However, to find out the limit of the pitch angle for which the theoretical equation can be used, the experiments were carried out using the range 0° to 45° .

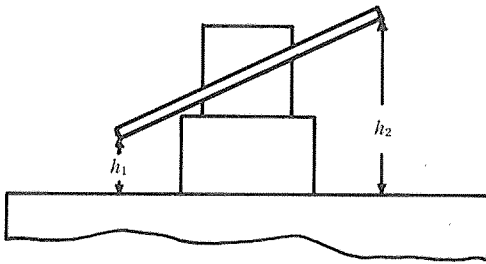


Fig. 6. Measurement of pitch angle.

4. Experimental Results and Discussion

4.1 Effects of the Pitch Angle

The results obtained by the experimental method which was mentioned in section 3.3 are

shown in Fig. 7. The value of the added polar moment of inertia is represented by the percentage of the polar moment of inertia in the air. The theoretical value of the added polar moment of inertia is obtained from equation (9) and in the numerical calculations, the part of a blade being divided into 15 equal parts, is calculated using the experimental results and the coefficients of the propeller. As seen in Fig. 7, the more the pitch angle increases, the more the experimental and theoretical values of the added polar moment of inertia increase. Namely, in the range between the pitch angle 0° and 45° , the experimental value becomes 1.4% to 33.6% and the theoretical value becomes 0% to 97.3%. Though the value of the added polar moment of inertia is theoretically 0% when the pitch angle is 0° , it appears that the experimental value is about 1.4%. This, as shown in Fig. 5, is due to the effect of the thickness of

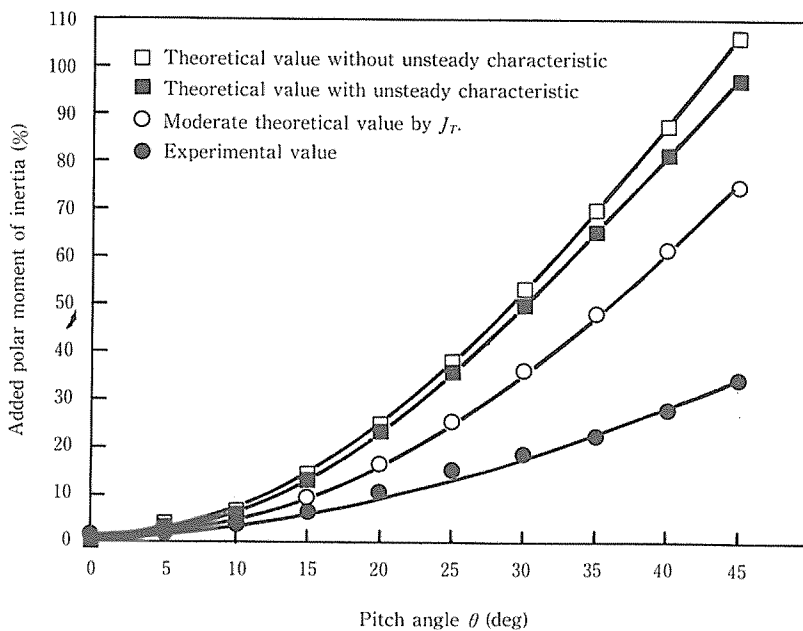


Fig. 7. Theoretical and experimental values of the added polar moment of inertia at each pitch angle.

the blade and the effect of the nuts which fix the blade on the boss.

Next, comparing the theoretical value with the experimental value, it is evident that the former is always greater than the latter. Especially, the difference increases rapidly after the pitch angle reaches 15° , and when it exceeds 20° , the theoretical value becomes several times greater than the experimental value. Consequently, it may be concluded that the difference between the theoretical and experimental values increases with elevated pitch angle. The rate of increase of the added polar moment of inertia at each pitch angle is shown in Fig. 8. As regards theoretical value, as the pitch angle increases, so does the rate of increase of the added polar moment of inertia. Notably the rate of increase rises linearly up to about pitch angle 30° , and when it exceeds 30° , it decreases slightly. As is clear from equation (9), this is the reason why the added polar moment of inertia is simply expressed by $\sin\beta(r)$. In com-

parison with the theoretical value, the experimental value has a tendency to increase slightly up to pitch angle 25° . If it exceeds 25° , it shows a decreasing tendency. It is clear from this tendency that the two-dimensional oscillating wing theory is the most adequate as a method to evaluate accurately the effects of added water. That is, as can be seen from equations (6) and (8), the value of the added polar moment of inertia is determined by the lift. If the pitch angle increases, the lift increases, and the added polar moment of inertia naturally increases. But in practice, the lift of wing increases linearly to about 20° and if it exceeds 20° , it decreases rapidly. Though the effect of the property of the lift is expressed in these results, it is not considered in the theoretical equation. Therefore, the great difference between the theoretical value and the experimental value appears when the pitch angle exceeds 25° .

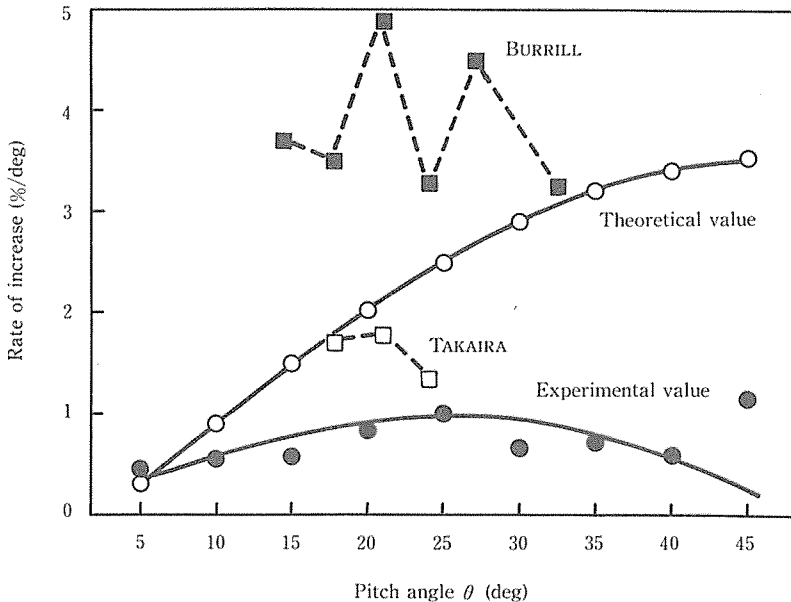


Fig. 8. The rate of increase of the added polar moment of inertia at each pitch angle.

In order to confirm the reliability of these experimental results, both BURRILL⁶⁾ and TAKAIRA'S³⁾ experimental results are shown in the same Figure. In their experiment they made a torsional vibration model using the series model propellers. The added polar moment of inertia was obtained from the torsional frequency without rotating the propellers. Consequently, their experimental method is different from our method. Comparing the results of this study with TAKAIRA'S results, though it is insufficient to compare them because of the narrow experimental range of the pitch angle, it can be seen that their results have nearly the same tendency as these results. But, as BURRILL'S experimental errors are great, it would be ineffective to compare his results with those of this study. It can be concluded that the results are influenced by the errors of each propeller's form because of using the series model propellers.

4.2 Unsteady Characteristic

As shown in equation (1), the lift of the wing under vibration in steady flow is the sum of the lift without steady flow (first part of the equation) and the unsteady lift due to the circulation produced by steady flow round a wing (second part, which contains $C(\nu)$). Therefore, when there is unsteady flow, the factor $\{1+2G(\nu)/\nu\}$ is added to the added polar moment of inertia I_{pw} . $G(\nu)$ and ν are expressed as equation (2). As shown in Fig.9, $2G(\nu)/\nu$ is always a negative value, so I_{pw} becomes smaller than the value which is in steady flow. ν is the dimensionless parameter which shows an unsteady characteristic, and also is determined by the chord length of a blade, frequency and velocity of flow (rotational speed of a propeller).

Furthermore, equation (10) can be derived from the relationship between the resonance rotational speed of a propeller Ω and the natural frequency ω in the torsional vibration of a marine propulsion shaft.⁹⁾

$$n\Omega = \omega \quad (10)$$

n : the order of torsional vibration

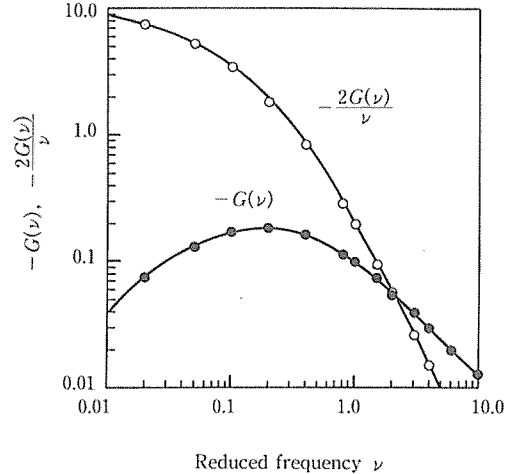


Fig. 9. Value of the unsteady characteristic.

where the velocity of flow $V(r)$ at the radial position r can be expressed as

$$V(r) = \frac{r\Omega}{\cos\beta(r)} \quad (11)$$

Rearranging equation (2) using equations (10) and (11), the reduced frequency ν , at the radial position r is

$$\nu_r = \frac{Bn\cos\beta(r)}{2r} \quad (12)$$

Therefore, in a state of resonance which is under torsional vibration, the value of the unsteady characteristic can be determined by the order of torsional vibration n , the radial position of a propeller r and the chord length of a blade B .

The calculation of the value of the unsteady characteristic for this model is demonstrated below. In this apparatus, the torque from a D.C. servo-motor provides the propeller shaft with rotational motion through the reduction gear (gear ratio 1 : 4), and also becomes a timevarying force of torsional vibration through the universal joint. Namely, the torque changes eight times during one rotation of a propeller shaft: n is equal to 8. The results of this calculation are shown in Fig. 10. From these results, it can be seen that the more

the radial of a propeller r increases, the more $|2G(\nu)/\nu|$ increases. The black marks in the figure indicate the values of the unsteady characteristic of I_{pw} (equation (9)) at each pitch angle. Therefore, the value of the unsteady characteristic $2G(\nu)/\nu$ can be represented by the value at the radial position of a propeller $0.7R$. $2G(\nu)/\nu$ of a propeller with a constant pitch angle can be represented by the value at the radial position $0.7R$. We tried to find out whether $2G(\nu)/\nu$ of a propeller whose pitch

angle is radially changed, such as a fixed pitch propeller, can be represented in the same way. The propellers used in the first paper of this study are used again as fixed propellers. The principal specifications of these propellers are given in Table 2. As a result of our investigations, as shown in Fig. 11, it can be seen that $2G(\nu)/\nu$ of the fixed pitch propeller is represented by the value at the radial position $0.7R$.

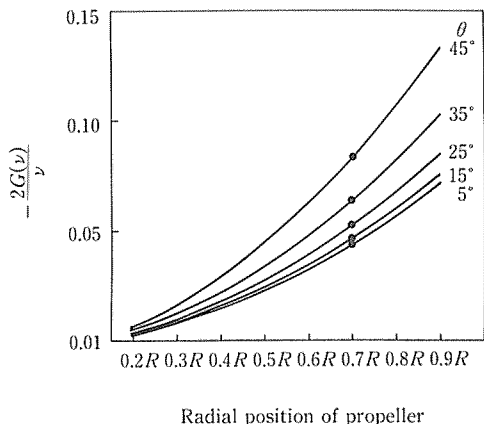


Fig. 10. Variation of the unsteady characteristic $\frac{2G(\nu)}{\nu}$ with the radial position for each pitch angle θ .

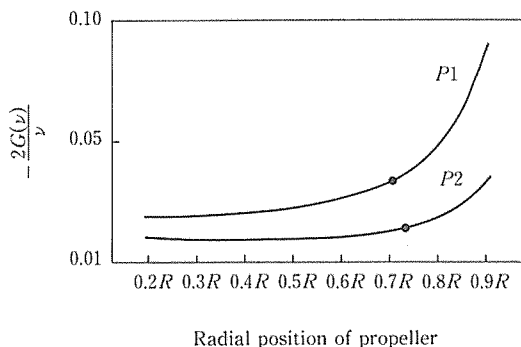


Fig. 11. Variation of the unsteady characteristic $\frac{2G(\nu)}{\nu}$ with each radial position of the propeller $P1$ and $P2$.

Table 2. Principal specifications of the propeller $P1$ & $P2$

Propeller	$P1$	$P2$
Number of blades	3	3
Diameter (cm)	32.8	33.0
Pitch ratio	0.979	0.948
Boss ratio	0.123	0.139
Exp. area ratio	0.343	0.517
Direction of turn	Right	Right
Mass (kg)	1.404	2.699
Aspect ratio	2.781	1.846

4.3 The Coefficient of the Three-dimensional Correction

The value of the added polar moment of inertia obtained from the two-dimensional oscillating wing theory is calculated to be greater than the experimental value. Especially a three-dimensional correction is necessary for the theoretical value because the difference between the theoretical and experimental values increases with elevated pitch angle. The coefficient of the three-dimensional correction J_T obtained from the lift line theory, as mentioned in the previous paper, is used as a three-dimensional correction. The relationship between J_T and pitch angle is shown in Fig. 12. KUMAI's²⁾ coefficient of the three-dimensional correction obtained from the potential theory is shown at the same time. KUMAI's coefficient is obtained using his table and the aspect ratio which is given by projecting the propeller used in this experiment on a plane at each pitch angle. J_T is almost equal to KUMAI's value. These values have a tendency to increase slightly with elevated pitch angle.

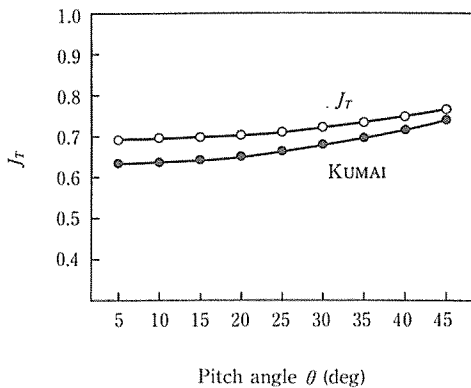


Fig. 12. Variation of the coefficient of the three-dimensional correction J_T with pitch angle.

As the two-dimensional theoretical value is always greater than the experimental value, it is corrected by the coefficient of the three-dimensional correction J_T . The curve which is shown with \circ in Fig. 7 is the theoretical value corrected by J_T . Though the theoretical value which is corrected by J_T becomes almost equal to the experimental value up to about pitch angle 25° , when the pitch angle exceeds approximately 25° , the difference between these becomes gradually greater, so the validity as a theoretical equation disappears. However the operational range of the pitch angle for the controllable pitch propeller is about -20° to $+20^\circ$. Therefore, the corrected theory using the coefficient of the three-dimensional correction is very effective for the controllable pitch propeller.

5. Concluding Remarks

The experimental investigation was carried out using a controllable pitch propeller to evaluate the effects of pitch angle on the added polar moment of inertia. As a result, the following can be understood:

1. The experimental value of the added polar moment of inertia rises from 0% to 32.3% as the pitch angle increases from 0° to 45° . Therefore, it is necessary to consider the effects of pitch angle when designing a propeller shaft on which a controllable pitch propeller is mounted.
2. In the practical range of pitch angle for a controllable pitch propeller, the theoretical value corrected by the coefficient of the three-dimensional correction J_T is almost equal to the experimental value, so it is very effective for the design of a controllable pitch propeller shaft system.
3. In a state of resonance which is under torsional vibration, the value of the unsteady characteristic can be determined by the order of the torsional vibration, the radial position of the propeller and the chord length of a blade.
4. The value of the unsteady characteristic can be represented by the value at the radial position of a propeller $0.7R$.

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プロペラの付加水質量並びに付加水
慣性モーメントに関する研究—II
可変ピッチプロペラにおけるピッチ角の影響

竹内正明・大崎栄喜・今 義英

可変ピッチプロペラの場合、運転条件によってピッチ角が変化するために、付加水質量や付加水慣性モーメントの大きさは、それぞれ変化する。特に付加水極慣性モーメントはピッチ角によって大きく影響を受ける。そこで本論文では、ピッチ角だけを変化させることができる模型プロペラを製作し、ピッチ角の影響について実験的検

討を行なった。その結果、可変ピッチプロペラを装備する推進軸系の設計においては、ピッチ角の影響を考慮する必要があること、および2次元振動翼理論は、3次元修正することによって、付加水極慣性モーメントの計算に十分有効であることなど明らかとなった。