A Trial for the Estimation of the Fishing Mortality of a Migrating Fish Procession. ※

By

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Introduction

Utilizing a period during which a fish procession passes through a fishing ground, namely, the so-called "fishing season", extension of that fishing ground and its corresponding catch, rate of fishing mortality can be estimated under such assumptions by the method mentioned below. The author confines himself in the present paper to exposition of principle of his method and to its application to a narrow fishing ground, but considerably valuable informations may be obtained by the application of this method to whole fishing ground of a specified species.

Basic Assumptions

At first we assume that arrangement of a fish procession maintains invariability over the whole area of a fishing ground and that this fish shoal travels at a constant speed from beginning to end.

Secondly it is assumed that magnitude of a fish procession decreases gradually at a constant rate of short-term fishing mortality f. Moreover, if natural mortality can be considered as negligible, rate of total diminution may be equal to the rate of fishing mortality.

At last, let us denote a common divisor for both dimensions of a fishing area and of a fish procession as Δ, then these dimensions may be described as $p\Delta$ and $q\Delta$ respectively while p and q can be estimated from pattern of fishing season at each part of a fishing ground.

And we denote the magnitude of the whole fish procession as $\phi = \sum_{i=1}^{q} \phi_i$, where ϕ_i corresponds to the magnitude of a fish sub-shoal involved in the i-th section of the fish procession which is divided equally into q sections.

On the basic assumptions mentioned above, we shall discuss the problem, for convenience' sake, by dividing it into two cases, namely, A) $p \geqq q$ and B) $p < q$.

Procedure in the Case A

The pattern of a fishing season at each part of a fishing ground in this case is shown in Fig. 1. It seems to be convenient to discuss separately after assorting

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this case into three occasions, i, ii and iii as follows :

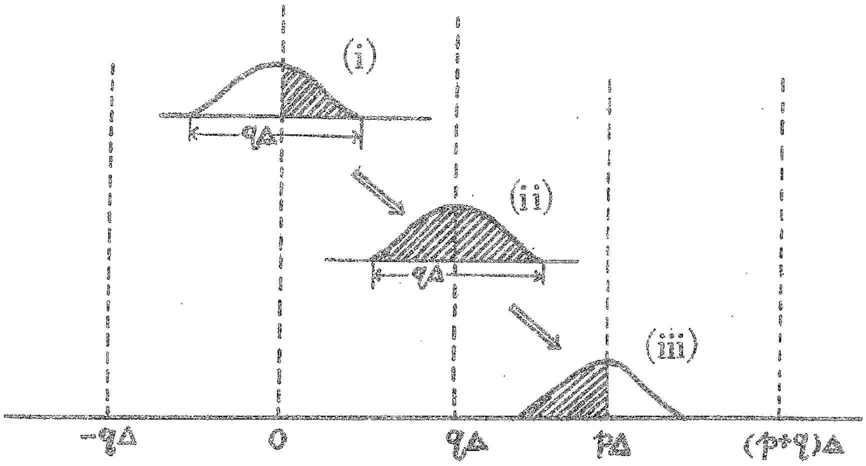

Fig. 1. Showing the transition of the portion confronted by the fishing operations (shaded). (p > q).

i) First occasion corresponds to a case in which the foremost section of a fish procession lies in an interval $o \sim q\Delta$ of a fishing ground. Only the sections of a fish shoal lying in the interval $o \sim q\Delta$ can be confronted with fishing operations. And, when the foremost section of this procession lies in an interval between $(k-1)\Delta \sim k\Delta$, an equation for amount of catch will be derived as follows :

$$c(k) = \sum_{i=1}^{k} f(1-f)^{(k-i)} \phi_i, \tag{1.1}$$

where $1 \leqq k \leqq q$.

And we may obtain the next equation showing the relation between two successive catch amounts,

$$c(k) = (1-f) \cdot c(k-1) + \phi_k. \tag{1.2}$$

Let us denote $C_1 = \sum_{k=1}^{q} c(k)$ and sum up from $k=1$ to $k=q$ both the sides respectively, then we obtain

$$fC_1 = f\phi - (1-f) \cdot c(q). \tag{1.3}$$

ii) In an occasion in which the top section of a fish procession lies in an interval $q\Delta \sim p\Delta$ (Fig. 1—ii), the whole shoal will be fished evenly. Thus an equation for catch is derived as follows :

$$c(q+k) = \sum_{i=1}^{q} f(1-f)^{q+k-i} \phi_i, \tag{1.4}$$

where $1 \leqq k \leqq p-q$, and from (1.4) we obtain

$$c(q+k) = (1-f) \cdot c(q+k-1). \tag{1.5}$$

So that, operating similarily as the occasion i, we obtain

$$fC_2 = (1-f) \cdot c(q) - (1-f) \cdot c(p), \tag{1.6}$$

where $C_2 = \sum_{k=1}^{p-q} c(q+k)$.

In case $p=q$ the equation (1. 6) is not necessary.

iii) At last when the first section lies in the interval $p\Delta \sim (p+q)\Delta$, the sections lying in the interval $p\Delta \sim (p+q)\Delta$ are excluded from the object of the fishing operations. So that

$$c\ (p+k) = \sum_{i=k+1}^{q} f\ (1-f)^{p+k-i}\ \phi_i, \tag{1.7}$$

where $1 \leqq k \leqq q$, and

$$c\ (p+q) = (1-f) \cdot c(p+k-1) - f\ (1-f)^p \phi_k. \tag{1.8}$$

So we can derive an equation as follows;

$$fC_3 = (1-f) \cdot c(p) - f\ (1-f)^p \phi, \tag{1.9}$$

where $C_3 = \sum_{k=1}^{q-1} c\ (p+k)$.

In case $p>q$, from (1.3), (1.6) and (1.9) we may obtain two equations as follows :

$$fC = f\phi\ \{\ 1 - (1-f)^p\},$$

or

$$\phi = C/\{\ 1 - (1-f)^p\}, \tag{1.10}$$

and

$$f = \{\ c\ (q) - c\ (p)\ \} / \{C_2 + c\ (q) - c\ (p)\}, \tag{1.11}$$

where $C \equiv C_1 + C_2 + C_3$.

When p equals to q, $c\ (q) = c\ (p)$, thus from (1.1) and (1.3) two equations may be derived as follows :

$$\phi = C' / \{\ 1 - (1-f)^p\}, \tag{1.10'}$$

where $C' \equiv C_1 + C_3$,

and

$$C_3 + (1-f)^p \cdot C_1 = c\ (p)\ \{\ 1 - (1-f)^p\}\ (1-f)/f. \tag{1.12}$$

If f is sufficiently small, we can consider that $(1-f)^p \doteqdot 1 - pf$, so that

$$f = \{C_1 + C_3 - p \cdot c(p)\}\ /p\ \{C_1 - c(p)\}.$$

Procedure in the Case B

We may perceive the pattern of relation among the catch amounts gained in each part of a fishing ground from Fig.2. And it is convenient to divide this case into three occasions.

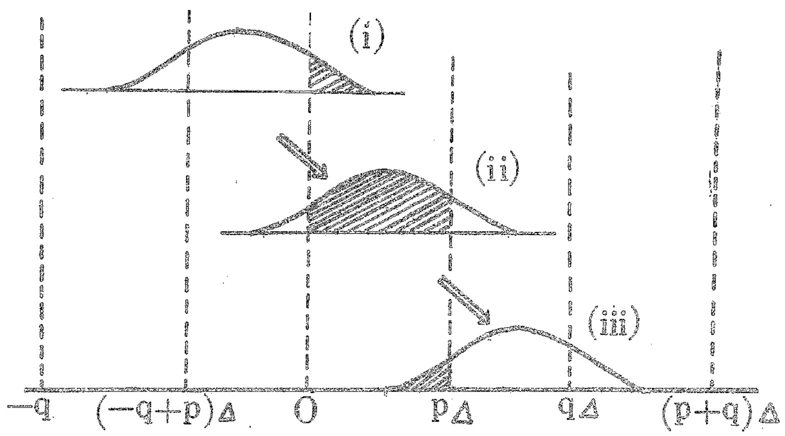

Fig. 2. Showing the transition of the portion confronted by the fishing operations (shaded). (p > q).

i) In the first occasion in which the foremost section lies in an interval o~pΔ (Fig. 2—i), a similar equation as (1.1) may be deduced as follows;

$$c(k) = \sum_{i=1}^{k} f\,(1-f)^{k-i}\,\phi_i, \tag{2.1}$$

where $1 \leqq k \leqq p$, with the exception of exchanging the symbol p for q. And let us denote as $\phi\,(o) = \sum_{i=1}^{p} \phi_i$, then, omitting midway procedure, we obtain

$$fC_1 = f\cdot\phi(o) - (1-f)\cdot c\,(p). \tag{2.2}$$

ii) When the first section lies in an interval pΔ~qΔ (Fig. 2—ii), only the sections lying in the interval o~pΔ can be confronted with the fishing operations and the residuary sections are free from the fishing. Thus

$$c\,(p+k) = \sum_{i=1}^{p} f\,(1-f)^{p-i}\,\phi_{k+i}, \tag{2.3}$$

where $1 \leqq k \leqq q-p$. So we obtain an equation as follows :

$$c\,(p+k) = (1-f)\cdot c(p+k-1) - f\,(1-f)^{p}\phi + f\phi_{p+k}. \tag{2.4}$$

So that

$$fC_2=(1-f)\ \{c(p)-c(q)\} - f(1-f)^{p}\phi(1)+f\cdot\phi(2), \tag{2.5}$$

where $\phi\,(1) = \sum_{i=1}^{q-p}\phi_i$, $\phi\,(2) = \sum_{i=1}^{q-p}\phi_{p+i}$ and $C_2 = \sum_{k=1}^{q-p} c\,(p+k)$.

iii) At last, when the first section lies in an interval qΔ~(q+p)Δ (Fig. 2—iii), an equation similar to (1.7) can be derived as follows :

$$c\,(q+k) \sum_{i=1}^{p-(k-1)} f\,(1-f)^{p-i}\,\phi_{q-p-1+k+i}, \tag{2.6}$$

where $1 \leqq k \leqq p$, and from (2.6) we can conclude that

$$c\,(q+k) = (1-f)\cdot c(q+k-1) - f\,(1-f)^{p}\phi_{q-p+k-1}. \tag{2.7}$$

So we can obtain an equation as follows :

$$fC_3 = c(q+1) - f\,(1-f)^{p}\phi(3), \tag{2.8}$$

where $\phi\,(3) = \sum_{i=2}^{p+1}\phi_{q-p+i-1}$.

From four ϕ (r)'s an equation will be deduced as follows :

$$\phi\,(0) + \phi\,(2) = \phi\,(1) + \phi\,(3) \equiv \phi. \tag{2.9}$$

In this case (p<q), we have necessity of assuming previously the type of arrangement of a fish procession, or deducing the type from other data by any proper procedure. If the type of arrangement is already known, then we can calculate the ratio $\phi(r)/\phi$, using the known values p and q. So, let us denote as $\phi\,(0) = \lambda_1\phi$ and $\phi\,(1) = \lambda_2\phi$, then we come to the following equations from (2.2), (2.5) and (2.8), viz.,

$$fC_1 = f\lambda_1\,\phi - (1-f)\cdot c(p),$$
$$fC_2 = f\,(1-\lambda_1)\,\phi - f\,(1-f)^{p}\lambda_2\,\phi + (1-f)\ \{c\,(p) - c\,(q)\}\ ,$$
$$fC_3 = \qquad - f\,(1-f)^{p}(1-\lambda_2)\,\phi + c\,(q+1).$$

From these three equations we can derive two equations as follows :

$$fC = f\ \{1-(1-f)^{p}\}\ \phi - (1-f)\cdot c(q) + c\,(q+1) \tag{2.10}$$

and

$$f\cdot(1-f)^{p} = [\ \{\ (1-\lambda_2)\ C_1+(1-\lambda_2)\ C_2+\lambda_2 C_3\}\ f$$

$$+ (1-f)(1-\lambda_2)\cdot c(q)] + f(1-f)^p\lambda_2\cdot c(q+1)$$
$$+ f^2C_3 - f\cdot c(q+1) = 0. \qquad (2.11)$$

It is generally troublesome to solve the equation of the (p+2)-order of f. Then, if f is a considerably small number, we may deduce from (2.11) the next equation, namely,

$$p\{(C_1+C_2) - \lambda_2(C_1+C_2-C_3)\} f^2$$
$$-[\{(C_1+C_2+C_3)-\lambda_2(C_1+C_2-C_3)-(1-\lambda_2)c(q)\}$$
$$-p\{c(q)-\lambda_2(c(q)-c(q+1))\}] f$$
$$-(1-\lambda_2)\{c(p) - c(q+1)\} = 0. \qquad (2.12)$$

So we can calculate the values of ϕ by substituting f from (2.12) into the equation (2.10).

Actual Example

Among three relations between p and q, the most frequent and practical one may occur in the case of $p<q$. So we bring up for discussion the sardine fishery by the drift-net along the coast of Niigata Prefecture facing the Japan Sea. From the spring to the early summer the sardines migrate northwards off the coast of this area. Used data are obtained from the General Report on the Surveys of the Tsushima Stream, No. 1, published in December, 1953 by the Niigata Prefectural Fisheries Experimental Station.

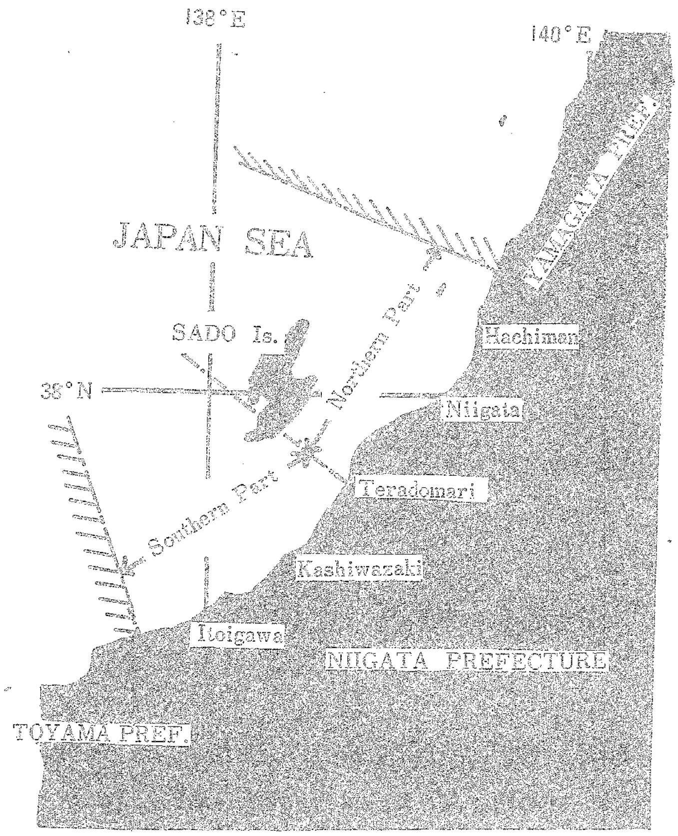

Fig. 3. Map of fishing area.

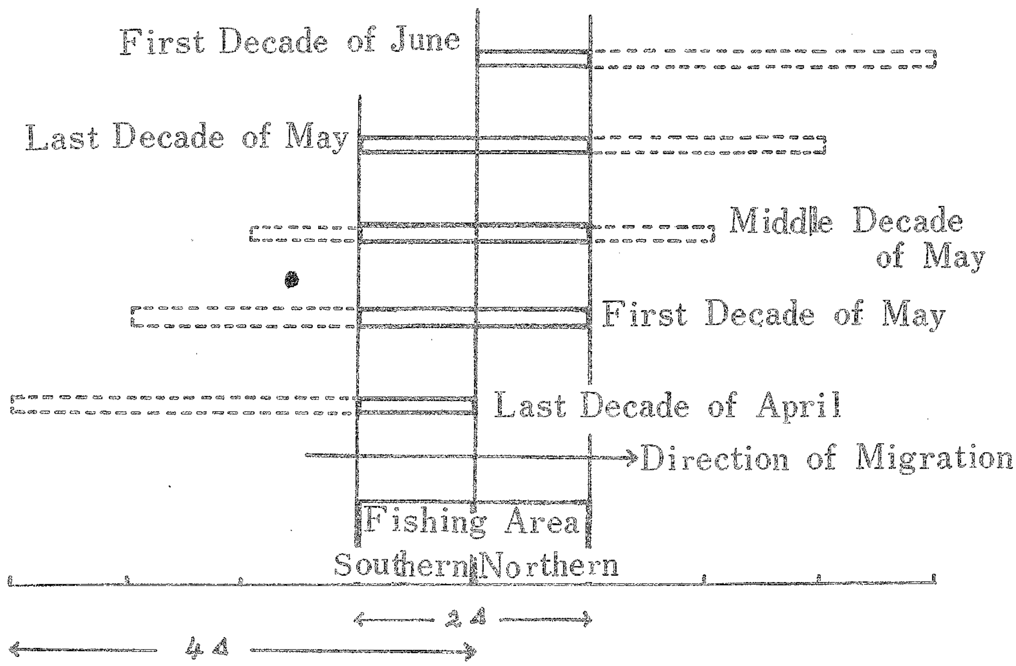

Fig. 4. p and q at the sardine fishery by the drift-net off Niigata Pref. (1953)

The sardine fishery by drift-net off the Niigata Prefecture seemed to continue from the last decade of April to the first decade of June in 1953. And it can be safely said that the pattern of fishery is distinguished between the northern half and the southern half of this area, the border line set at Teradomari. In other words, the fishing season of the northern half extends from the first decade of May to the first decade of June and the season in the southern half continues from the first decade of April to the last decade of May. The extension of the coast line of each part may be regarded as approximately equal.

Table 1. Numbers of fishes in the northern half and the southern half of Niigata Prefecture, caught by the drift-nets in the season of 1953 ($\times 10^3$). The sardine, *sardinia melanosticta.*

Period / Fishing ground	April	May			June	Total
	Last Decade	First Decade	Middle Decade	Last Decade	First Decade	
Southern Half	348.7	3082.4	1937.7	906.7	—	6275.5
Northern Half	—	1365.0	1575.7	2332.7	693.4	5966.8
Total	348.7	4447.4	3513.4	3239.4	693.4	12242.3
Diagonal Sum	—	1713.7	4658.1	4270.4	1600.1	—

And, as distinct from Fig. 3, $p=2$ and $q=4$. So, let us denote the magnitude of each sub-shoal corresponding to Δ as ϕ_i and define as $\check{c}_i = f\phi_i$ in the southern half and $c'_i = f(1-f)\phi_i$ in the nothern part, then we may obtain the following results from Table 1.

$$\ddot{c}_0 = 348.7 \times 10^3, \qquad \dot{c}'_0 = 1365.0 \times 10^3,$$

$$\check{c}_1 = 3082.4 \times 10^3, \qquad \dot{c}'_1 = 1575.7 \times 10^3,$$

$$\ddot{c}_2 = 1937.7 \times 10^3, \qquad \dot{c}'_2 = 2332.7 \times 10^3,$$

$$\check{c}_3 = 906.7 \times 10^3, \qquad \dot{c}'_3 = 639.4 \times 10^3,$$

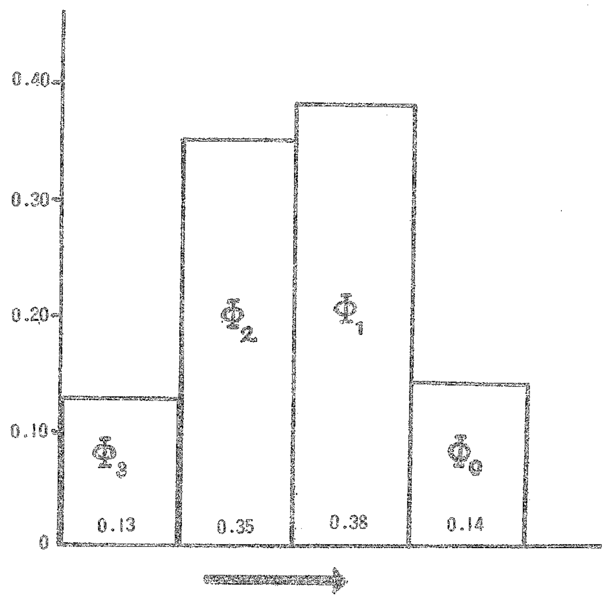

Fig. 5. Pattern of ϕi.

So that $\phi_0 : \phi_1 : \phi_2 : \phi_3 = 0.14 : 0.38 : 0.35 : 0.13$ (Fig. 5).
As p=2 and q=4, it follows

$$\lambda_1 = 0.14 + 0.38 = 0.52, \qquad 1-\lambda_1 = 0.48,$$

$$\lambda_2 = \quad 0.14, \qquad 1-\lambda_2 = 0.86.$$

And it results that

$$C_1 = \dot{c}'_0 + \ddot{c}'_0 + \ddot{c}_1 = 4786.1 \times 10^3,$$

$$C_2 = \check{c}'_1 + \check{c}_2 + \check{c}'_1 + c_3 = 6752.8 \times 10^3,$$

$$C_3 = \check{c}'_3 = c\,(q+1) = 693.4 \times 10^3,$$

$$c\,(p) = \dot{c}'_0 + \dot{c}_1 = 4447.4 \times 10^3,$$

$$c\,(q) = \dot{c}'_2 + \check{c}_3 = 3249.4 \times 10^3.$$

Therefore, regarding as $(1-f)^p \doteqdot 1-pf$, the results are as follows :

$$f^2 - 0.1065f - 0.0081 = 0.$$

So that

$$f = 0.158.$$

Thus, the rate of diminution during the season may be estimated as follows :

$$\{ f+f(1-f)+f(1-f)^2+\cdots\} = 0.487,$$

or, the rate of survival migrating out from this fishery ground is 0.513.

So,

$$\phi^* = C/0.487 = 25,138.2 \times 10^3 \fallingdotseq 25 \times 10^6.$$

Conclusion

Of the assumptions used as the basis of this trial, the first one appears to be not essential, for the irregularity of the arrangement in which the fish shoal migrates or the evenness of the migrating speed is not worth our considerations if the emigration or imigration through each boundary of a fishing area proceed continuously and smoothly.

The second assumption may be inadequate to some actual cases. In the coasting fisheries which are in the habit of aiming at the migrating pelagic fishes, generally speaking, towards the openning of fishing season a few boats begin to scout for a fish shoal for trial, the number of boats tends to increase as the season advances and to follow the main shoal within a certain limit and finally the fishing intensity comes to drop rapidly at a certain time, being perhaps a little behind the time when all fish shoals have gone out from this area. So if the number of fishing boats increases in proportion to the amount of fishes at hand, it may be safely assumed as f=const. In the last half of the fishing season, if anything, the assumption f=const. will fall to be inadaptable. In other report the author will discuss the case f=f (t).

As mentioned previously, the limit of operations of the fishing boats tends to surpass the area covering those base ports. So one shoud take into account the actual conditions for the limitation of the fishing area.

This method may be considerably useful only when a fish shoal travels in parallel with the coast line. Still, when a fish shoal returns backward after staying for some days in a fishing ground, the same procedure may be adaptable with some additional considerations.

In this report the author omitts the considerations on the recruitment due to the birth of young fishes, the imigrations of two or more fish processions coming through quite different paths and the estimation of total amount of fish population which contains the fish stock being confronted to the fishery and another fish stock which will be capable of being fished during the period under consideration. On these cases the author will present other discussions in the future.

Reference

1) ANON : 1953. General Report on the Surveys of the Tsushima Stream, No.1 Dec. 1953. (published by the Niigata Prefectural Fisheries Experimental Station), (in Japanese).

* ϕ does not mean magnitude of total population existing in a more extensive fishing ground involving the above-mentioned area but represents only magnitude of a fish shoal available for fishery in that limited fishing ground during the season under consideration. Accordingly, by extending this method of estimation over all the neighbouring fishing grounds successively, approximate magnitude of total population may be calculated.